

Implementation and Verification of Nonlinear Spring–Damper Restraints in OpenFOAM’s Six-DOF Rigid-Body Motion Framework

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Abstract. This work presents the development of new OpenFOAM® libraries that extend existing linear spring–damper models to incorporate nonlinear translational and rotational restraints in a 6-DOF rigid-body analysis case. The implementation allows arbitrary stiffness and damping relationships to be defined through analytical models, such as Duffing and Rayleigh formulations, or via user-specified interpolation tables. The libraries are embedded directly into the 6-DOF rigid body motion solver, enabling fluid–structure interaction simulations considering nonlinear dynamic oscillations, without relying on external structural solvers. Numerical verification against central difference schemes, analytical models, and preCICE-coupled OpenFOAM® – CalculiX simulations demonstrates good agreement for linear and nonlinear single-DOF systems, with only minor deviations in amplitude response. Application cases, including wind-induced oscillations of airfoils and galloping of a 3:2 prism, further illustrate the ability of the libraries to capture nonlinear structural dynamics within realistic flow conditions. The proposed implementation extends the existing approach and provides an efficient and flexible framework for investigating nonlinear aeroelastic phenomena and energy harvesting concepts, modelling the dynamic system as a nonlinear 6-DOF system in OpenFOAM®.

1. Introduction

Computational modelling of fluid–structure interaction (FSI) problems is a remarkably challenging endeavour that has attracted considerable attention from the scientific community over the years. Previous studies have outlined both the scientific and practical significance of this research field [1]. One active area of investigation involves wind-induced excitation problems, where high Reynolds number flows interact with light and flexible structures undergoing large-amplitude oscillations.

Wind-excited energy harvesters represent a canonical application example where kinetic energy of the wind is transformed into oscillatory mechanical energy due to resonant excitation and subsequently into electrical power by transducer technologies. Common ambient excitation sources like wind typically contain energy distributed across multiple frequencies [2]. Conventional single degree of freedom (DOF) linear vibration harvesters operate efficiently only near resonance, i.e., when the excitation frequency matches the natural frequency of the harvester. Consequently, they perform poorly under broadband or unpredictable excitations [3]. Several strategies have been proposed to overcome this limitation. For instance, Tang and Zuo [3] developed control strategies to tune the resonant frequency dynamically, while Trimble [4] investigated multi-degree of freedom designs to provide multiple resonances. Although these methods improve adaptability and output power, they also introduce drawbacks, such as energy overhead for control systems or increased complexity and reduced robustness in multi-DOF configurations.

A common alternative to enhance performance of energy harvesters is the design of dynamic systems featuring nonlinear characteristics [5]. Unlike linear systems, nonlinear ones do not possess a fixed natural frequency, which might be dependent, for instance, on the displacement of the system. Depending on the

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system's parameters this makes nonlinear systems more resilient to changes in external excitation; and as a result, they maintain strong performance across a broad range of conditions, as shown by Gendelman et al. [6] and Sapsis et al. [7]. Wind-excited energy harvesters featuring nonlinear stiffness have therefore been extensively investigated [5, 8] through analytical models and experiments, given their potential for superior performance in terms of adaptability and energy output.

Nevertheless, the underlying FSI mechanisms governing the aeroelastic interplay between the flow and nonlinear dynamical systems remain insufficiently explored through numerical simulations. From a computational perspective, the standard approach in FSI modelling is to represent the structure as a linear dynamical system, typically with a single DOF [9, 10]. Simulating nonlinear dynamical systems, however, remains difficult due to the lack of specific tools tailored to such strongly coupled problems. Within the open-source computational fluid dynamics (CFD) environment, the SixDoF module in OpenFOAM® only considers linear dynamics systems. Alternatively, libraries like preCICE [11, 12] have enabled coupling between fluid solvers such as OpenFOAM® and structural solvers like CalculiX. While this methodology permits addressing highly general cases, including multi-DOF structural systems, it requires proficiency in multiple software environments and often entails a significant computational cost characteristic of partitioned approaches.

To overcome these limitations, the present work proposes an extension of existing OpenFOAM® libraries, specifically the `linearSpring` and `linearAxialAngularSpring` modules, to incorporate nonlinear translational and rotational springs and dampers directly within the OpenFOAM® framework. This provides a straightforward and efficient way to investigate nonlinear structural systems under fluid excitation without relying on external structural solvers. The implementation is verified against analytical and numerical results for undamped and damped 1DOF dynamic systems, followed by the application to wind-excited oscillators featuring nonlinear characteristics. This approach not only enables efficient and accurate numerical simulations of nonlinear FSI problems considering six DOF rigid systems, but also bridges the gap between analytical/experimental studies and high-fidelity computational modelling.

The remainder of the paper is organised as follows: Section 2 introduces the mathematical formulation of nonlinear dynamical systems, including stiffness and damping models, as well as their step-by-step numerical integration. Section 3 describes the implementation of the proposed nonlinear spring-damper libraries within the OpenFOAM® framework. Section 4 presents verification cases comparing the new implementation against established methods. Section 5 and 6 provide application examples to demonstrate the capabilities of the developed libraries in state of the art fluid-structure interaction problems. Finally, Section 7 outlines the main conclusions of this piece of research.

2. Formulation

2.1. Linear and nonlinear dynamical system: general formulation.

The motion of a single DOF mass-spring-damper dynamical system is defined by Eqn. 1, where a linear relationship between an arbitrary dynamic load and the displacement, velocity and acceleration of the system is considered [13]:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t). \quad (1)$$

In the above equation, $x(t)$ represents the time-dependent displacement of the lumped mass m out from the balance position, while $\ddot{x}$ is the acceleration and $\dot{x}$ is the velocity, while t is the time. In Eqn. 1, the term $m\ddot{x}(t)$ represents the inertial force, the second term, $c\dot{x}(t)$, corresponds to the damping force, with c denoting the structural damping coefficient, while the third term, $kx(t)$, is the restoring force exerted by the spring, where k is the stiffness of the spring. In Eqn. 1, the constant values of k and c are inherent to the linear nature of the system, leading to a linear differential equation of motion that can be solved by standard procedures. Eqn. 1 can be also written in terms of equivalent forces as:

$$F_I(t) + F_D(t) + F_S(t) = F(t), \quad (2)$$

where $F_I(t)$, $F_D(t)$, and $F_S(t)$ are the forces exerted on the dynamical system due to inertia, damping and stiffness at any point in time, respectively, which may be linear or nonlinear terms. In the next subsections a range of nonlinear formulations for nonlinear stiffness and damping are introduced.

2.2. Nonlinear stiffness.

It has been mentioned in the introduction that there are several cases of dynamical systems whose behaviour depart from the linear stiffness relationship. One classical nonlinear stiffness model, profusely applied in vibration energy harvesters is the Duffing oscillator [14], whose force-displacement relationship, that is the stiffness force in Eqn. 2, is given by:

$$F_S(t) = Ax(t) + Bx^3(t). \quad (3)$$

Depending on the values of the parameters A and B , the system may behave as a linear one ($A > 0$ and $B = 0$), as a symmetric bistable system ($A < 0$ and $B > 0$), as a hardening spring ($A > 0$ and $B > 0$), or as a softening spring ($A > 0$ and $B < 0$).

Another example of a dynamical system featuring mechanical nonlinearities is a cantilever wrapping around a curved surface as it vibrates [2, 15]. In Fig. 1, a conceptual scheme of the cantilever beam and the contact surfaces is presented. Depending on the values of the geometry parameters presented in Fig. 1, different nonlinear relationships for the tip displacement of the cantilever and the exerted force can be obtained.

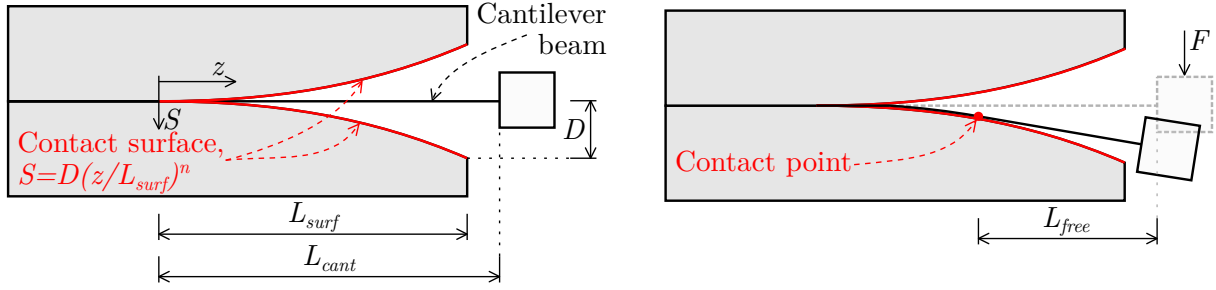


Figure 1. Sketch of the cantilever beam featuring contact surfaces. L_{cant} is the length of the cantilever, L_{surf} is the length of the contact surface from the beam root following the z axis, L_{free} is the distance in the z axis from the contact point to the end of the undeflected cantilever, D is the gap between the surface end and the undeflected cantilever, and n is the exponent value defining the shape of the contact surface.

Figure 2 depicts the force-displacement curve obtained with the parameters reported in Tab. 1, that is used in the application case in section 6. It must be noted that there is no analytical solution for obtaining these curves, that represent a cantilever beam wrapping around curved surfaces (Fig. 1), and they must be solved numerically following the procedure depicted in [2], which is a modification of Timoshenko's design [16] and considers: that the cantilever only slides a small amount along the surface (friction forces are discarded), only one moving part and the contact surfaces prevent excessive deflections preventing yielding. The work by [2] is a real application of nonlinear springs to an energy harvester solution, therefore it is considered appropriate to use the same constitutive relationship for the nonlinear spring in our simulations and the application case.

Table 1. Parameter values of the cantilever beam featuring contact surfaces for the calculation of the relationship between force and displacement.

L_{cant} [m]	L_{surf} [m]	D [m]	n	D [N/m ²]	I [m ⁴]
3.0	2.652	0.073	4.518	2.1×10^{11}	4.459×10^{-8}

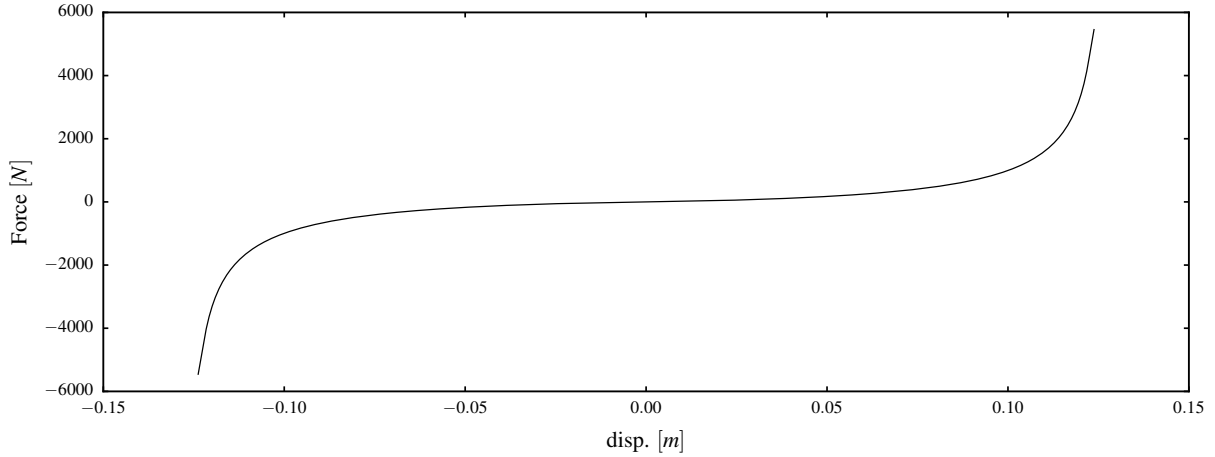


Figure 2. Nonlinear relationship between force and displacement for the 3:2 prism application case in section 6.

2.3. Nonlinear damping.

In most of the cases, the damping of dynamical systems is modelled by means of a direct relationship with the velocity of the system through viscous damping c , although most structural systems feature a certain level of nonlinear characteristics, which often is acceptable to neglect to facilitate the analysis [17]. However, in other cases, such as in some energy harvesting applications, the nonlinear damping should be explicitly considered in the dynamic analyses [2]. In general, the nonlinear damping force tends to increase with the excitation level. For a single DOF, a general nonlinear damping force term in the dynamic equilibrium equation may be written as [18]:

$$F_D = c_1 \dot{x}(t) + h(\dot{x}(t)), \quad (4)$$

where c_1 is the linear damping and $h(\dot{x}(t))$ is a function representing the nonlinear term, for which several relationships have been proposed.

For instance, a widespread formulation for non linear damping is the cubic expression proposed by Rayleigh [18–20]:

$$F_D = c_1 \dot{x}(t) + c_2 \dot{x}^3(t). \quad (5)$$

In the specific field of broadband bistable energy harvesters, Mann [21] adopted the following expression for the nonlinear damping term:

$$F_D = c_a \dot{x}(t) + c_b \dot{x}^2(t) \dot{x}(t), \quad (6)$$

where c_a and c_b are the linear and nonlinear damping coefficients respectively. Evidently, there may be cases whose nonlinear relationship between the damping force and the velocity cannot be properly modelled by relatively simple analytical expressions like the ones in Eqns. 5 and 6. In those cases a tabular input representing the explicit relationship between damping force and the system's velocity may be required to properly simulate the system's behaviour.

2.4. Multi-DOF nonlinear dynamical system.

The expressions introduced above can be generalised for a multi-DOF dynamical system to obtain the dynamic equilibrium equation in matrix form as:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + [\mathbf{H}(\dot{\mathbf{q}}(t), \mathbf{q}(t))] \dot{\mathbf{q}}(t) + [\mathbf{K}(\mathbf{q}(t))] \mathbf{q}(t) = \mathbf{Q}(t). \quad (7)$$

Where $\mathbf{M}$ and $\mathbf{C}$ are the linear mass and damping matrices, $\mathbf{H}$ is the nonlinear damping matrix, $\mathbf{K}$ is the nonlinear stiffness matrix, $\mathbf{Q}$ is the generic load vector, and $\mathbf{q}$ is the generalised displacement vector.

2.5. Step-by-step integration of nonlinear systems.

Standard approaches for solving the dynamic equilibrium equations such as mode superposition are based on the superposition principle, which can only be applied to linear systems. In the frame of fluid-structure interaction problems comprising nonlinear dynamical systems, the most efficient strategy [13]

is to apply a time marching approach, adopting small time steps that enables considering the dynamic properties of the system inside each of the time steps as constant. It is to note that the properties of the nonlinear system may change from one time instant to another due to the changes in displacement and velocity.

In the application cases presented herein, the Newmark- β method has been applied for the step-by-step time integration of the nonlinear dynamic equilibrium equation with parameters $\gamma=1/2$ and $\beta=1/4$, corresponding to the constant acceleration case [22]. That implementation of the method is unconditionally stable for linear systems, and in some cases for softening nonlinear systems [23].

The degree of nonlinearity can be defined within a time step as the ratio of the final stiffness or damping to the initial stiffness or damping, and in the next equation the degree of nonlinearity for the stiffness is written as:

$$\mu_{i+1} = \frac{k_{i+1}}{k_i}, \quad (8)$$

where k_i and k_{i+1} are the stiffness at the beginning and the end of the i^{th} time step.

According to [24], even for hardening nonlinear systems, the constant acceleration implementation of the Newmark- β method delivers stable results for very small time steps that delay numerical instability by keeping the degree of nonlinearity μ_{i+1} close to one and consequently the spectral radius also close to one. Furthermore, small time steps typically produce lower truncation errors. The impact of the time step size in the stability and accuracy of the step-by-step integration of the dynamic response of several nonlinear systems is addressed in section 4 using the newly developed libraries presented herein.

2.6. Approximation of the time-dependent stiffness and damping forces.

In the subsections above, the nonlinear stiffness and damping models have been defined based on relatively simple analytical expressions. However, it is to note that there exist more complex cases, such as cantilever beams whose displacement is constrained by curved surfaces, mentioned in subsection 2.2, whose nonlinear behaviour cannot be modelled adopting simple analytical expressions. Aiming at providing nonlinear stiffness and damping libraries in OpenFOAM[®] with general capabilities, arbitrary nonlinear relationships between stiffness force and displacement, and damping force and velocity, may be defined by the user employing text files as input in the dynamicMeshDict for the OpenFOAM[®] libraries presented herein. These text files contain pairs of points "displacement" vs "restoring force" $(\delta_i, F_{Si}, i = 1, \dots, n)$ and/or "velocity" vs "damping force" $(\dot{\delta}_i, F_{Di}, i = 1, \dots, n)$ for any of the six DOF associated to the rigid body motion of the dynamical system. In this subsection, the procedure implemented for approximating the values of the stiffness and damping forces for each time step along the step-by-step integration of the dynamic equation is explained.

At any time t , the restoring force acting on a single DOF system may be evaluated based on the interpolation over the characteristic discrete displacement vs restoring force curve defined by n pairs of points $(\delta_i, F_{Di}, i = 1, \dots, n)$. These pairs define a piecewise linear representation of the nonlinear restoring force of the system. In this context, restoring force refers to forces or moments, and displacement to translations or rotations. The restoring force is then computed as:

$$F_S(t) = f(\delta(t)), \quad (9)$$

where $f(\delta(t))$ is the result of the interpolation or extrapolation operation over the user-defined relative displacement vs restoring force pairs $\{(\delta_i, F_{Si}, i = 1, \dots, n)\}$ at the considered time step.

In the newly developed libraries, linear and cubic spline interpolations are available (see subsections 3.4 and 3.5). The same procedure can be applied for the approximation of the damping force at time t .

The proposed approach enables modelling 6-DOF rigid body systems with arbitrary nonlinear stiffness and damping behaviour.

3. Computational implementation in the OpenFOAM[®] libraries

The standard OpenFOAM[®] distribution includes a 6-DOF rigid body motion framework that allows the simulation of moving bodies interacting with a fluid. Within this framework, the motion of a rigid body can be constrained by mechanical restraints such as springs or dampers. However, the default implementation only supports linear spring–damper relationships, where stiffness and damping coefficients remain constant during the simulation. Many structural systems of practical interest exhibit nonlinear behaviour and therefore require more flexible formulations. To address this limitation, new OpenFOAM[®] libraries were developed to support nonlinear stiffness and damping relationships directly within the solver. The implementation follows a modular architecture composed of three main components.

The first component consists of the main nonlinear restraint libraries `nonLinearSpring` and `nonlinearAxialAngularSpring`, which describe translational and rotational nonlinear spring–damper behaviour. The second component is the `interExtrapolationTable` library, which performs linear interpolation combined with controlled extrapolation for tabulated nonlinear relationships. The third component is the `InterExtrapolateSplineXY` library, which provides cubic spline interpolation together with smooth extrapolation outside the tabulated range.

This modular structure separates the physical modelling of nonlinear restraints from the numerical procedures used to evaluate nonlinear stiffness and damping curves. As a result, the restraint classes focus on computing displacements, velocities and rotations of the rigid body, while the interpolation libraries are responsible for evaluating nonlinear relationships defined through tabulated data. This design improves the flexibility, maintainability and extensibility of the implementation.

3.1. Integration with the 6-DOF motion solver.

The developed libraries are fully integrated within the OpenFOAM 6-DOF rigid body motion framework. In this framework the motion of a rigid body is governed by Newton’s equations and is updated at every time step based on the forces and moments acting on the body. Restraints are implemented through classes derived from the base class `sixDoFRigidBodyMotionRestraint`. By inheriting from this class, the nonlinear restraints can be incorporated into the existing motion solver without modifying the core solver structure.

At runtime, the restraint model is selected through the configuration dictionary `dynamicMeshDict`. The OpenFOAM runtime selection mechanism registers the nonlinear libraries so that they can be instantiated based on the user-defined configuration. This allows the nonlinear restraints to be activated simply by specifying the corresponding model name in the case setup.

The code fragment shown in Lst. 1 illustrates the registration of the nonlinear rotational restraint within the OpenFOAM[®] runtime selection table.

```

1  defineTypeNameAndDebug(nonlinearAxialAngularSpring, 0);
2
3  addToRunTimeSelectionTable
4  (
5      sixDoFRigidBodyMotionRestraint,
6      nonlinearAxialAngularSpring,
7      dictionary
8  );

```

Listing 1. Inclusion of the nonlinear rotational restraint within the OpenFOAM[®] runtime selection table.

This approach allows the solver to dynamically select the appropriate restraint model during the simulation without requiring recompilation.

3.2. Nonlinear translational spring library.

The `nonLinearSpring` library implements nonlinear translational spring–damper behaviour between an anchor point and an attachment point located on the rigid body. At every time step the solver computes the position of the attachment point based on the current rigid body motion. The relative distance between the attachment point and the anchor location determines the spring extension.

The direction of the spring force is defined by the unit vector connecting the anchor point and the attachment point. The magnitude of the spring displacement is obtained by subtracting the rest length of the spring from the current distance between the two points.

The displacement calculation implemented in the library is shown in the code fragment presented in Lst. 2.

```
1 vector r = restraintPosition - anchor_;
2 scalar magR = mag(r);
3 r /= (magR + VSMALL);
4
5 scalar displacement = magR - restLength_;
```

Listing 2. Displacement calculation.

Once the spring displacement has been calculated, the restoring force is evaluated using the stiffness model selected by the user. Several alternative formulations are supported.

The simplest option corresponds to a linear spring model, where the restoring force is proportional to the displacement. A more general formulation is the Duffing nonlinear stiffness model, which introduces a cubic displacement term as shown in Lst. 3.

```
1 springForce = A.*displacement + B.*pow3(displacement);
```

Listing 3. Code of the Duffing nonlinear stiffness model.

This model allows the simulation of both softening and hardening nonlinear stiffness behaviour, depending on the sign and magnitude of the cubic coefficient.

For more complex structural systems, the nonlinear stiffness relationship may be defined through tabulated displacement – force pairs provided by the user. In this case the restoring force is evaluated through interpolation using the `interExtrapolationTable` library, as shown in Lst. 4.

```
1 springForce = stiffnessTablePtr_()(displacement);
```

Listing 4. Calculation of the spring force when using tabulated stiffness.

Alternatively, cubic spline interpolation can be used to evaluate the nonlinear stiffness curve, providing a smoother representation of the force–displacement relationship. The code for this implementation is shown in Lst. 5

```
1 springForce =
2     InterExtrapolateSplineXY
3     (
4         displacement,
5         *springX_,
6         *springY_,
7         springExtrapType_
8     );
```

Listing 5. Calculation of the spring force when using cubic spline interpolation.

3.3. Nonlinear rotational spring library.

The `nonlinearAxialAngularSpring` library extends the nonlinear restraint formulation to rotational motion. In this case the restoring moment depends on the angular displacement of the rigid body around a specified axis.

The angular displacement is determined by comparing the reference orientation of the rigid body with its current orientation. The rotation is projected onto the specified axis and the corresponding angular displacement is computed, as shown in Lst. 6.

```
1 scalar theta = acos(min(oldDir & newDir, 1.0));
```

Listing 6. Calculation of the angular displacement.

The restoring moment is then evaluated using the selected stiffness model. When tabulated stiffness data are used, the moment magnitude is obtained through interpolation, as indicated in Lst. 7.

```
1 springMag = stiffnessTable.(theta);
```

Listing 7. Calculation of the restoring moment when using tabulated stiffness.

The total restoring moment acting on the rigid body includes both the spring contribution and the damping contribution and is applied along the rotation axis. This is implemented as shown by the fragment of code in Lst. 8

```
1 restraintMoment = -(springMag + dampingMag)*a;
```

Listing 8. Calculation of the total restoring moment, considering spring and damping contributions.

3.4. Linear interpolation and controlled extrapolation.

When nonlinear stiffness or damping relationships are defined through tabulated values, interpolation must be used to determine the corresponding force value for the current displacement or velocity. For this purpose the `interExtrapolationTable` library extends the standard OpenFOAM interpolation table functionality.

The library performs linear interpolation between neighbouring data points within the tabulated range. The interpolated value is computed using the slope between the two surrounding points, as shown in Lst. 9.

```
1 return
2 (
3     y0 + (y1 - y0)*(lookupValue - x0)/(x1 - x0)
4 );
```

Listing 9. Calculation of the interpolated values.

In addition to interpolation within the tabulated domain, the library also provides controlled behaviour when the lookup value lies outside the available data range. Several options are available, including warnings, clamping, repeating the data periodically, or performing linear extrapolation based on the slope of the nearest segment. The code for the linear extrapolation is shown in Lst. 10.

```
1 return
2 (
3     yN + (lookupValue - xN)*(yN - yNm1)/(xN - xNm1)
4 );
```

Listing 10. Calculation of values outside the available data range using linear extrapolation.

3.5. Spline interpolation with smooth extrapolation.

Although linear interpolation is sufficient for many applications, it may introduce discontinuities in the slope of the nonlinear force curve. To obtain smoother nonlinear relationships, the `InterExtrapolateSplineXY` library implements cubic spline interpolation.

In this method the interpolated value is computed using neighbouring points in the dataset, as shown in Lst. 11, producing a smooth curve across the interpolation range.

```

1  return
2  0.5 *
3  (    2*y1
4    + mu *
5      (
6        -y0 + y2 + mu *
7          (
8            (2*y0 - 5*y1 + 4*y2 - y3) + mu*(-y0 + 3*y1 - 3*y2 + y3)
9          )
10     )
11 );

```

Listing 11. Calculation of values using spline interpolation.

This interpolation method provides a continuous representation of nonlinear stiffness and damping curves and is particularly useful when the tabulated data originate from experimental measurements or detailed structural simulations and they are relatively sparse.

Overall, the developed libraries extend the OpenFOAM 6-DOF motion framework by enabling nonlinear stiffness and damping relationships to be defined analytically or through tabulated data. The separation between physical models and interpolation routines provides a flexible and efficient framework for simulating nonlinear structural dynamics directly within OpenFOAM®.

Figure 3 illustrates how the nonlinear translational (**nonLinearSpring**) and rotational (**nonlinearAxialAngularSpring**) restraints are embedded within the OpenFOAM® sixDoF motion framework. Both classes inherit the library **sixDoFRigidBodyMotionRestraint** base, ensuring compatibility with dynamic mesh solvers such as **sixDoFRigidBodySolver**. Each restraint can be configured with independent spring and damper components, allowing users to model a wide range of nonlinear restoring forces and dissipative effects. For the stiffness model, users may choose between a simple linear stiffness, a Duffing-type nonlinear stiffness, or a user-specified stiffness list. For the damping model, supported formulations include linear, Rayleigh, Mann, and tabulated damping laws. This versatile design provides flexibility in capturing complex structural dynamics by enabling different nonlinear models for translation and rotation within the same solver framework.

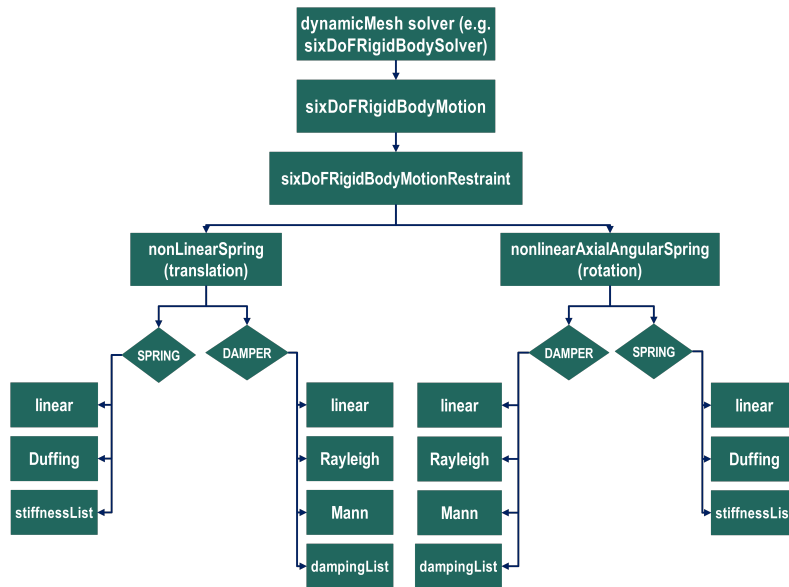


Figure 3. Integration of the nonlinear spring–damper libraries into the OpenFOAM® sixDoF framework, showing translational (**nonLinearSpring**) and rotational (**nonlinearAxialAngularSpring**) restraints with available spring and damper models.

3.6. Examples of how to use the newly developed libraries.

In Lsts. 12-14 examples of how to use the developed libraries for the translational spring are provided, while in Lst. 15 it is shown an example for a rotational spring.

```

1 restraints
2 {
3
4     verticalSpring
5     {
6
7         sixDoFRigidBodyMotionRestraint nonLinearSpring;
8         anchor          (0.25 0.007 0.125);
9         refAttachmentPt (0.25 0.007 0.125);
10        restLength      0;
11        modelType        Duffing;
12        A                4000; //linear term
13        B                200000; //cubic term
14        dampingModelType Mann; //Rayleigh;
15        c1               6; //linear term
16        c2               5000; //cubic term
17    }
18 }
19
20 }
```

Listing 12. Definition of a nonlinear translational spring using the Duffing model and a nonlinear damper using the Mann model.

```

1 restraints
2 {
3
4     verticalSpring
5     {
6         sixDoFRigidBodyMotionRestraint nonLinearSpring;
7
8         anchor          (0.25 0.007 0.125);
9         refAttachmentPt (0.25 0.007 0.125);
10        restLength      0;
11        modelType        stiffnessList;
12
13        stiffnessList
14        {
15            interpolationType linear; //cubicSpline;
16            outOfBounds      EXTRAPOLATE;
17
18            values
19            (
20                // (restoringForce displacement)
21                ( -1.0 -4000.0)
22                (  0   0)
23                (  1.0 4000.0)
24            );
25        }
26
27        dampingModelType linear;
28        c1               6;
29    }
30 }
```

Listing 13. Definition of a nonlinear translational spring using a table relating restoring force with displacement using linear interpolation and a linear damper.

```

1 restraints
2 {
3     verticalSpring
4     {
5         sixDoFRigidBodyMotionRestraint nonLinearSpring;
6
7         anchor          (0.25 0.007 0.125);
8         refAttachmentPt (0.25 0.007 0.125);
9         restLength      0;
10        modelType       stiffnessList;
11
12        stiffnessList
13        {
14            interpolationType linear;//cubicSpline
15            outOfBounds      EXTRAPOLATE;
16
17            values
18            (
19                //(restoringForce displacement)
20                ( 0.0    0.0 )
21                ( 1.0    4000.0 )
22            );
23        }
24
25        dampingModelType    dampingList;
26        listType            velocity;//displacement;
27
28        dampingList
29        {
30            interpolationType linear;//cubicSpline
31            outOfBounds      EXTRAPOLATE;
32
33            values
34            (
35                //(damperForce velocity) in case of selecting displacement
36                //                        in listType it would be
37                //                        (damperForce displacement)
38                ( 0.0    0.0 )
39                ( 1.0    2.0 )
40            );
41        }
42    }
43 }

```

Listing 14. Definition of a nonlinear translational spring and a nonlinear damper using a table relating force and displacement for the spring and force and velocity for the damper.

4. Numerical verification of nonlinear stiffness and damping libraries

All the verification cases reported next consider three different oscillators: freely decaying one DOF linear spring-damper system, undamped nonlinear stiffness one DOF systems and freely decaying one DOF nonlinear spring-damper systems. The time-dependent oscillations for the two first cases under consideration have analytical solutions, which enables a robust numerical verification of the newly developed libraries for nonlinear systems. Furthermore, in this section a sensitivity study for the dynamic response depending on the size of the time step adopted in the step-by-step integration of the dynamic equilibrium equation is reported, addressing accuracy and stability of the solutions. It must be noted that for the cases solved with OpenFOAM®, the flow domain surrounding the object under study must feature a finite volume mesh despite the analysis addressing only the structural dynamic response. In order to reduce to a negligible level the influence of flow effects in the resolution of the problem under

study, as the focus is in the structural system response, the fluid velocity was set to $1.0\text{e-}6$ m/s, the fluid density to $1.0\text{e-}6$ kg/m³ and the kinematic viscosity was set to $1.0\text{e-}12$ m²/s. In all the presented results the temporal evolution of the amplitudes and frequencies was calculated by means of a Hilbert transform. On the other hand, damping temporal evolution is calculated using an equivalent linearisation approximation of the non-linear dynamic system as proposed in [25]. This method only considers the slow modulation behaviours, obtaining the instantaneous damping ratio using the derivative of the natural logarithm of the instantaneous amplitude of the movement, and hence the model is sensitive to small amplitude values, producing certain numerical amplification in those cases. The verification cases that are reported in this section refer to single DOF dynamic systems, since these are canonical oscillators featuring analytical solutions that enable direct verification of the numerical solutions. Furthermore, this facilitates the physical interpretation of nonlinear dynamic properties, preventing the coupling between various DOF.

```

1  restraints
2  {
3      axialSpring
4      {
5          sixDoFRigidBodyMotionRestraint nonLinearAxialAngularSpring;
6
7          axis          (0 0 1);
8          dampingModelType    Rayleigh;//Mann;
9          c1      20;
10         c2      50;
11         referenceOrientation $orientation;
12
13         modelType      stiffnessList;
14
15         stiffnessList
16         {
17             interpolationType linear;//cubicSpline
18             outOfBounds      EXTRAPOLATE;
19
20             values
21             (
22                 (-0.1745329 -2459.578788)
23                 (0.0000000  0.000000)
24                 (0.1745329  2459.578788)
25             );
26         }
27     }
28 }

```

Listing 15. Definition of a nonlinear rotational spring and a nonlinear damper using a table relating moment and rotation for the spring and using the Rayleigh model for the damper.

4.1. Linear stiffness and damping. Numerical verification and time step sensitivity.

The objective of this verification case is to check the accuracy of the `nonLinearSpring` and `nonLinearAxialAngularSpring` libraries newly implemented on OpenFOAM® for cases featuring linear stiffness. In this manner, the overall reliability of the computational implementation of the new libraries is guaranteed.

For the translational case, the constant stiffness and damping properties (see Tab. 2) were provided in the newly developed `nonLinearSpring` library by introducing a list of points for stiffness and damping. In Fig. 4, the analytical solution is compared with the results provided by the new library on OpenFOAM® considering three different time steps: 7.2×10^{-5} s, 1.0×10^{-5} s and 1.0×10^{-6} s. The agreement with the analytical solution is remarkably good, with the larger time step providing results that slightly depart from the analytical solution. Furthermore, Tab. 3 shows the time-averaged total energy obtained for the analytical solution and the three simulations considering different time step sizes. Since viscously damped systems are non-conservative, the time-averaged total mechanical energy is evaluated as the mean value of the total mechanical energy over 20 cycles of oscillation. The comparison between the values of the

mean total mechanical energy for different time step sizes allows assessing the time step size impact in the time-integrated response. The results are consistent with the unconditional stability expected for the constant-average-acceleration Newmark scheme applied to a linear system.

Table 2. Mechanical properties of the single DOF system with linear translational spring. m is the mass of the system, k is the spring constant, c is the viscous damping of the system, x_0 is the initial position of the system and $\dot{x}_0$ is the initial velocity of the system.

m [kg]	k [N/m]	c [Ns/m]	x_0 [m]	$\dot{x}_0$ [m/s]
22.9	4000.0	6.0	0.0	2.65

Table 3. Comparison of the time averaged total energy ($\overline{E_{TOT}}$) and the root mean square error (RMSE) between the oscillation time-histories of the analytical solution and the ones obtained by OpenFOAM[®] considering different time step sizes, for the case of the translational spring with constant stiffness and damping.

	Analytical	Δt		
		7.2×10^{-5}	1.0×10^{-5}	1.0×10^{-6}
$\overline{E_{TOT}}$ [J]	25.331	26.710	25.463	25.344
RMSE [m]		2.8067×10^{-3}	2.7376×10^{-4}	2.7307×10^{-5}

A similar study has been conducted for a rotational linear system aimed at verifying the `nonLinearAxialAngularSpring` new library. The constant stiffness and damping (see Tab. 4) were provided to the library by means of a list depicting a linear relationship between angular rotation and moment for stiffness, and angular velocity and moment for damping, respectively. The results obtained for three different time steps are presented in Fig. 5 along with the analytical solution for the free vibration case. Again, a remarkable agreement between the analytical timeseries of the rotation and the numerical results was identified. As for the translational simulations, only the case featuring the largest time steps shows minor differences in the peak rotation amplitudes. Tab. 5 reporting the time-averaged total mechanical energy over the same 20 cycles of oscillation further supports the previous comment.

Table 4. Mechanical properties of the single DOF system with linear rotational spring. I is the inertia of the system, k is the spring constant, c is the viscous damping of the system, α_0 is the initial angular position of the system and $\dot{\alpha}_0$ is the initial angular velocity of the system.

I [kgm ²]	k [Nm/rad]	c [Nms/rad]	α_0 [rad]	$\dot{\alpha}_0$ [rad/s]
40.6452	8000.0	20.0	0.0	1.3519

Table 5. Comparison of the time averaged total energy ($\overline{E_{TOT}}$) and the root mean square error (RMSE) between the oscillation time-histories of the analytical solution and the ones obtained by OpenFOAM[®] considering different time step sizes, for the case of the rotational spring with constant stiffness and damping.

	Analytical	Δt		
		7.2×10^{-5}	1.0×10^{-5}	1.0×10^{-6}
$\overline{E_{TOT}}$ [J]	7.507	7.793	7.535	7.510
RMSE [rad]		8.418×10^{-4}	8.2303×10^{-5}	1.0619×10^{-5}

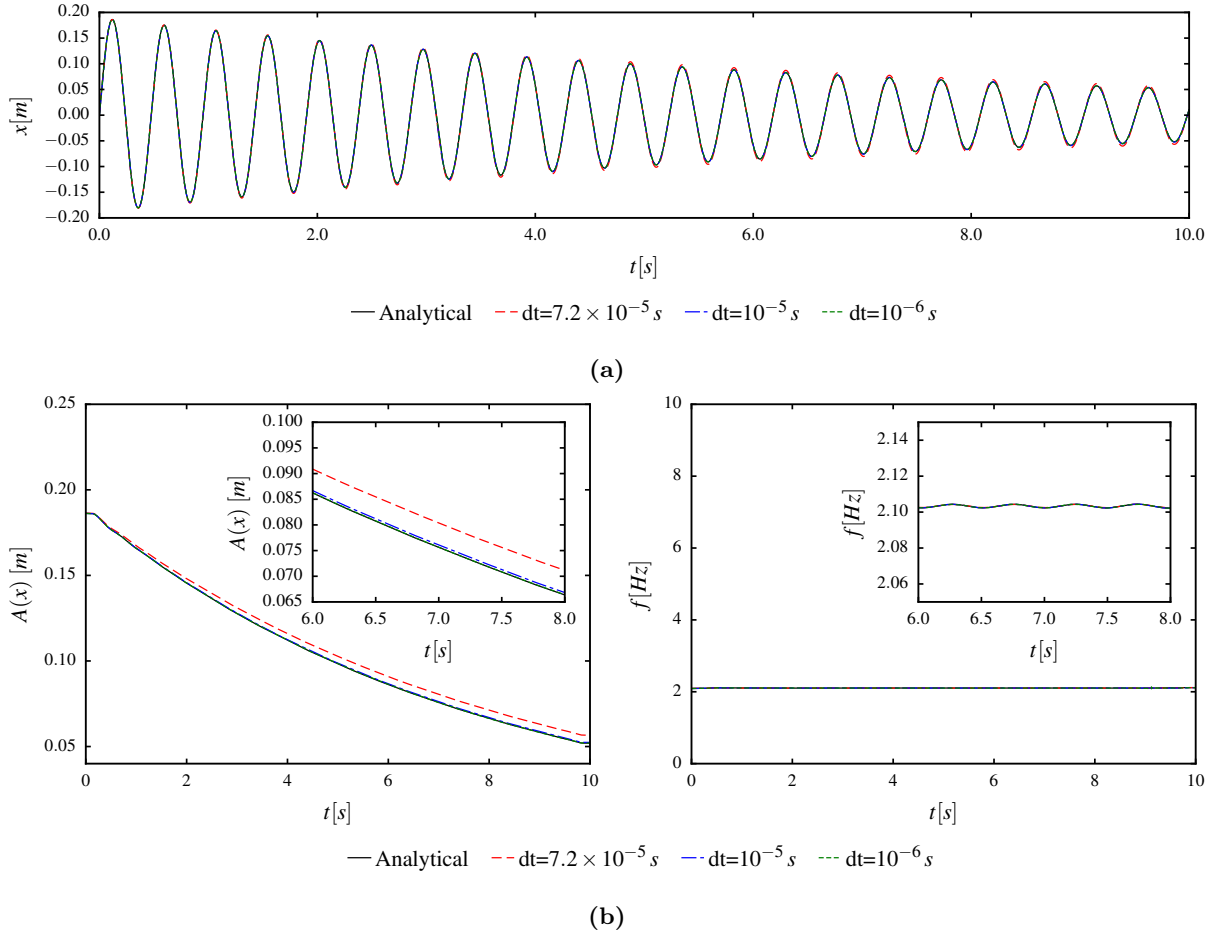


Figure 4. Translational linear stiffness and linear damping dynamic system response: analytical vs. `nonLinearSpring` library. a) Time-dependent free damped vibration and b) Time dependent oscillation amplitude (left) and frequency (right).

The number of time steps, considering their different sizes, for resolving one oscillation cycle for the translational and rotational spring cases reported above are presented in Tab. 6.

Table 6. Number of time steps needed for solving an oscillation cycle for the translational and rotational springs with constant stiffness and damping.

Δt [s]	$n(\Delta t)$	
	Translational	Rotational
7.2×10^{-5}	6604	6221
1.0×10^{-5}	47549	44791
1.0×10^{-6}	475489	447908

4.2. Nonlinear stiffness and zero damping. Numerical verification and time step sensitivity.

The objective of this section is to verify the `nonLinearSpring` and `nonLinearAxialAngularSpring` libraries for cases featuring nonlinear stiffness. To this end, the numerical results for conservative undamped 1 DOF cases behaving as Duffing oscillators in translational and rotational DOF were compared with the analytical solutions, whose closed form for this particular case is:

$$x(t) = x(0)cn\left(\sqrt{A + Bx(0)^2}t, \frac{Bx(0)^2}{2(A + Bx(0)^2)}\right), \quad (10)$$

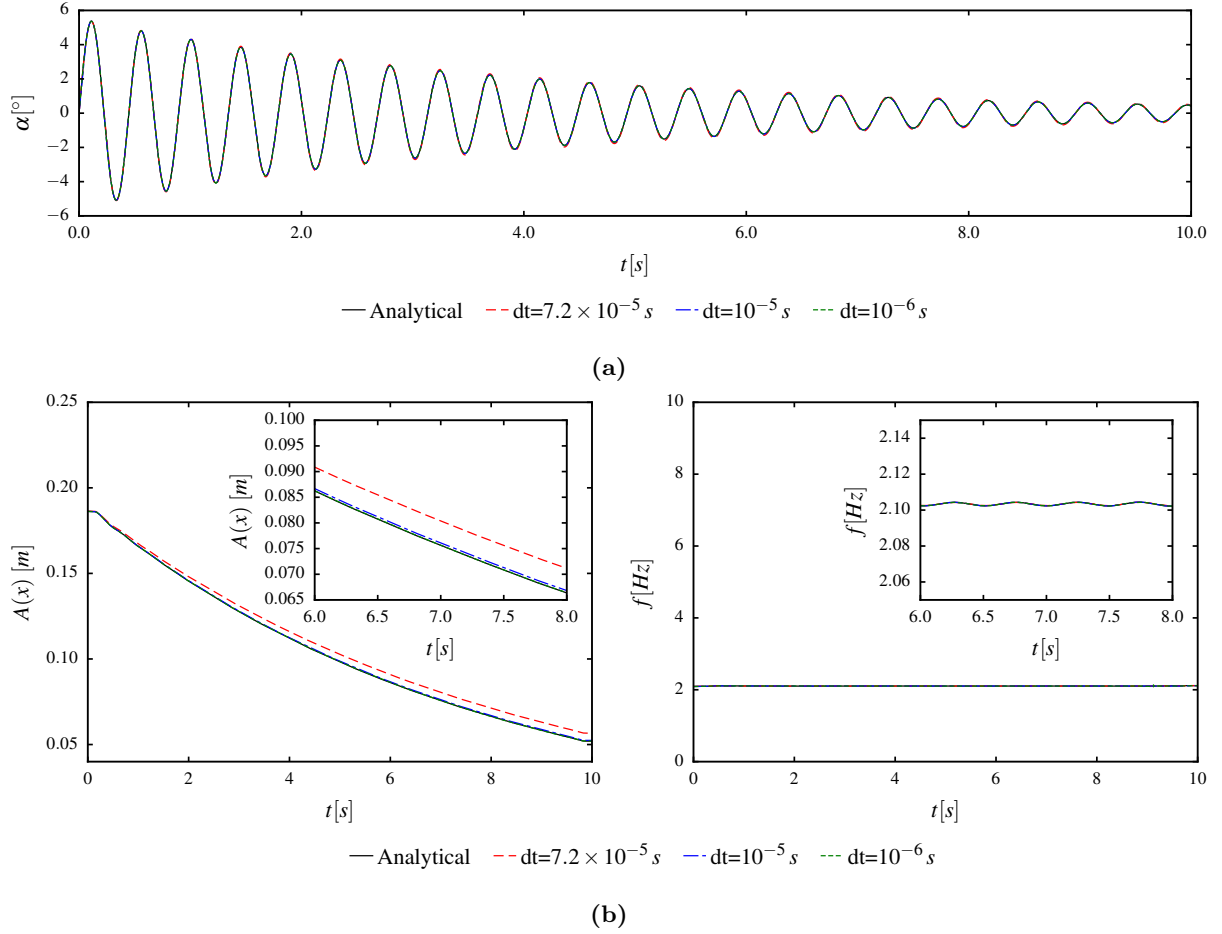


Figure 5. Rotational linear stiffness and linear damping dynamic system response: analytical vs. `nonLinearAxialAngularSpring` library. a) Time-dependent free damped vibration and b) Time dependent oscillation amplitude (left) and frequency (right).

where $x(0)$ is the initial amplitude of the oscillation, A and B are the parameters defining the Duffing model and cn stands for the elliptic cosine. Furthermore, the sensitivity of the dynamic response with the time step size was assessed.

For the translational undamped nonlinear case, the dynamic properties are reported in Tab. 7, including the parameters of the Duffing model. It was mentioned in subsection 2.5 that the constant acceleration implementation of the Newmark- β method is not unconditionally stable for nonlinear systems, but that short time steps enable keeping the degree of nonlinearity close to one, and therefore, delaying numerical instability. Fig. 6a shows how smaller time steps provided solutions closer to the analytical one, featuring a remarkable agreement with it. Furthermore, Fig. 6b presents a mild instability in the solution for the largest time step case, with peak amplitude and frequency slowly departing from the analytical solution as the simulation progresses. The results demonstrate the correct implementation of the `nonLinearSpring` library, providing solution for the dynamic equilibrium equation almost identical to the analytical solution for the time step 1.0×10^{-6} s, as supported by the values obtained for the mean total mechanical energy integrated over 10 s reported in Tab. 8. It is to note that for an undamped oscillator, the dynamical system is conservative.

The mechanical properties of the undamped nonlinear rotational case are reported in Tab. 9 including the parameters of the Duffing model. In Fig. 7, the results obtained for the undamped free rotation case applying the step-by-step method with three different time steps are reported, finding that smaller time steps (1.0×10^{-5} s and 1.0×10^{-6} s) deliver results almost identical to the analytical solution, while the largest time step shows mild instability, with the numerical solution departing from the analytical one as time advanced, as supported by Tab. 10. Again, the newly developed `nonLinearAxialAngularSpring` library was able to provide a solution for the integration of the dynamic equilibrium equation in full

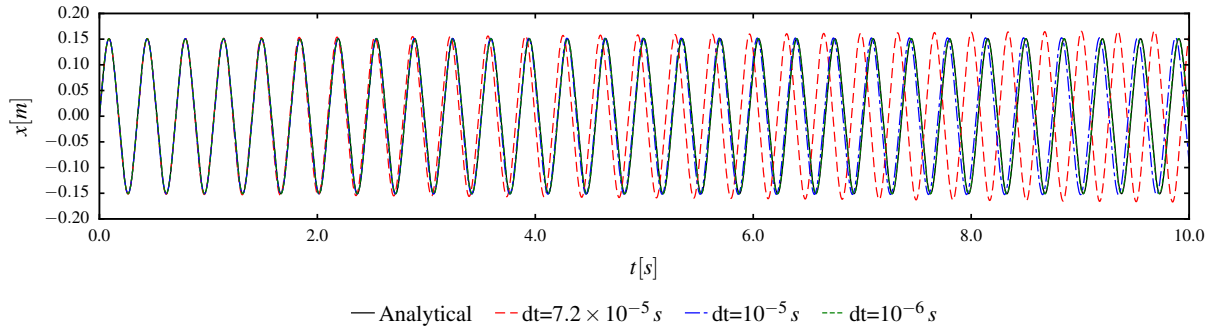
agreement with the analytical one.

Table 7. Mechanical properties of the undamped single DOF system with Duffing model. m is the mass of the system, A is the linear spring constant of the Duffing model, B is the nonlinear spring constant of the Duffing model, x_0 is the initial position of the system and $\dot{x}_0$ is the initial velocity of the system.

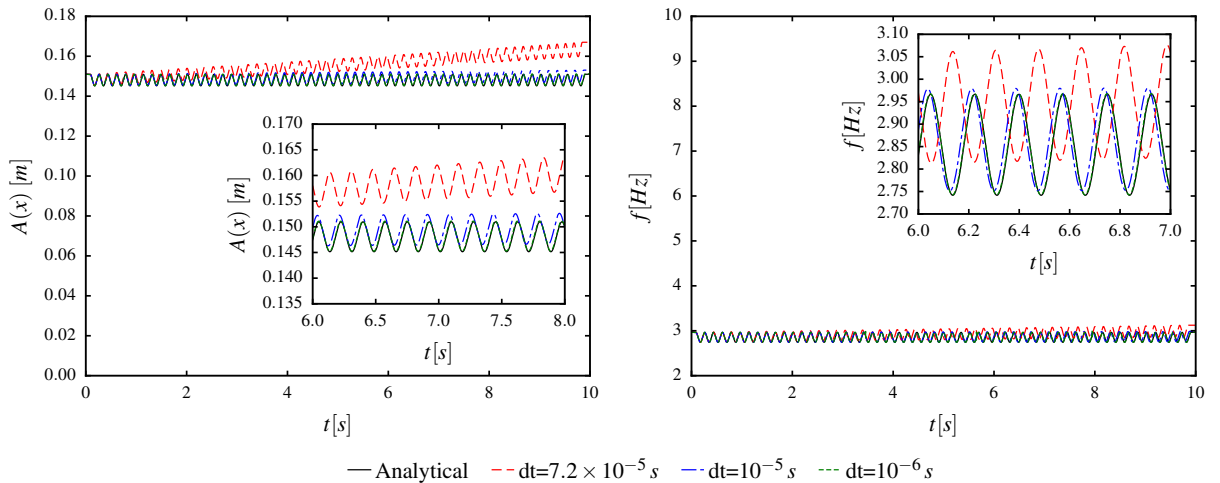
m [kg]	A [N/m]	B [N/m ³]	x_0 [m]	$\dot{x}_0$ [m/s]
22.9	4000.0	200000.0	0.0	2.5

Table 8. Comparison of the time averaged total energy ($\overline{E_{TOT}}$) and the root mean square error (RMSE) between the oscillation time-histories of the analytical solution and the ones obtained by OpenFOAM[®] considering different time step sizes, for the case of the translational spring with nonlinear stiffness and zero damping.

	Analytical	Δt		
		7.2×10^{-5}	1.0×10^{-5}	1.0×10^{-6}
$\overline{E_{TOT}}$ [J]	80.803	93.853	82.384	80.956
RMSE [m]		1.3562×10^{-1}	2.6200×10^{-2}	2.6179×10^{-3}



(a)



(b)

Figure 6. Translational nonlinear stiffness and undamped dynamic system response: analytical vs. `nonLinearSpring` library. a) Time-dependent free undamped vibration and b) Time dependent oscillation amplitude (left) and frequency (right).

Table 9. Mechanical properties of the single DOF system with nonlinear rotational spring. I is the inertia of the system, A is the linear constant of the Duffing model, B is the nonlinear constant of the Duffing model, α_0 is the initial angular position of the system and $\dot{\alpha}_0$ is the initial angular velocity of the system.

I [kgm ²]	A [Nm/rad]	B [Nm/rad ³]	α_0 [rad]	$\dot{\alpha}_0$ [rad/s]
40.6452	8000.0	200000.0	0.0	1.3519

Table 10. Comparison of the time averaged total energy ($\overline{E_{TOT}}$) and the root mean square error (RMSE) between the oscillation time-histories of the analytical solution and the ones obtained by OpenFOAM[®] considering different time step sizes, for the case of the rotational spring with nonlinear stiffness and zero damping.

	Analytical	Δt		
		7.2×10^{-5}	1.0×10^{-5}	1.0×10^{-6}
$\overline{E_{TOT}}$ [J]	38.446	42.029	38.914	38.492
RMSE [rad]		2.4502×10^{-2}	3.2207×10^{-3}	3.1890×10^{-4}

Based on the sensitivity studies reported in subsections 4.1 and 4.2, a time step of 1.0×10^{-6} s, that provides results almost coincident with the analytical solutions, has been selected for the simulations that are reported in subsection 4.3.

The number of time steps, considering their different sizes, for resolving one oscillation cycle for the translational and rotational spring cases reported above are presented in Tab. 11.

Table 11. Number of time steps needed for solving an oscillation cycle for the translational and rotational springs with nonlinear stiffness and zero damping.

Δt [s]	$n(\Delta t)$	
	Translational	Rotational
7.2×10^{-5}	4890	5784
1.0×10^{-5}	35207	41645
1.0×10^{-6}	352070	416449

4.3. Nonlinear stiffness and nonlinear damping. Numerical verification.

In this subsection the `nonLinearSpring` and `nonLinearAxialAngularSpring` libraries are used to solve the damped free vibration case of translational and rotational 1 DOF oscillators, featuring nonlinear behaviour both in the stiffness and damping forces. To verify the computational implementation of the new libraries, the OpenFOAM[®] solution for the damped free vibration of the nonlinear system is compared with the central finite differences solution (explicit direct integration method) of the nonlinear equilibrium equation for the same case, updating the stiffness and damping values at each time step. The time step used in the resolution of the case was 1.0×10^{-6} s, for the reasons stated in subsection 4.2.

The solution applying the central difference method consists in substituting the acceleration and velocity, for the linear cases or the ones applying the non linear relationships of stiffness and damping provided in section 2, by Eqs. 11 and 12 and solving the resultant equation for the function value at next time step.

$$\dot{x} = \frac{x(t_{i+1}) - x(t_{i-1}))}{2\Delta t}. \quad (11)$$

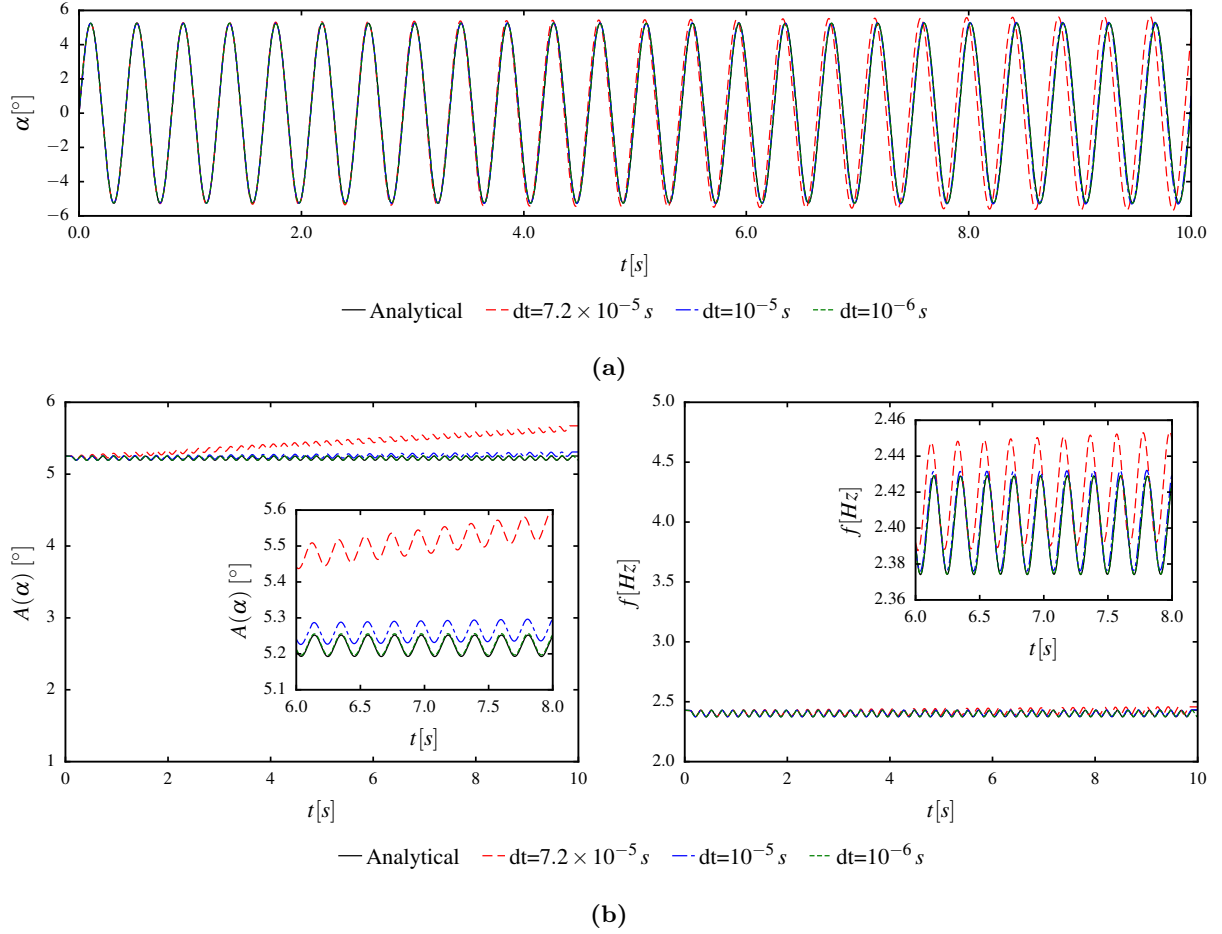


Figure 7. Rotational nonlinear stiffness and undamped dynamic system response: analytical vs. `nonLinearAxialAngularSpring` library. a) Time-dependent free undamped vibration and b) Time dependent oscillation amplitude (left) and frequency (right).

$$\ddot{x} = \frac{x(t_{i-1}) - 2x(t_i) + x(t_{i+1}))}{(\Delta t)^2}. \quad (12)$$

For the translational 1 DOF system, whose properties are reported in Tab. 12, the results are depicted in Fig. 8. In the figure, both approaches provide identical solutions, verifying the `nonLinearSpring` library implementation, particularly, its nonlinear damping implementation. In Fig. 8b, the time-dependent evolution of the damping is provided in the bottom left chart, showing strong modulation in damping over time. The translational case in subsection 4.1 (see Fig. 4) is the equivalent linearised case of the previously defined fully nonlinear case.

Table 12. Mechanical properties of the nonlinear single DOF system with Duffing model for stiffness and Mann model for damping. m is the mass of the system, A is the linear spring constant of the Duffing model, B is the nonlinear spring constant of the Duffing model, c_1 is the linear constant of the Mann model, c_2 is the nonlinear constant of the Mann model, x_0 is the initial position of the system and $\dot{x}_0$ is the initial velocity of the system.

m [kg]	A [N/m]	B [N/m ³]	c_1 [Ns/m]	c_2 [Ns ³ /m ³]	x_0 [m]	$\dot{x}_0$ [m/s]
22.9	4000.0	200000.0	6.0	5000.0	0.0	2.65

A similar case has been solved for a rotational 1 DOF fully nonlinear system (stiffness and damping). The dynamic properties of the system are reported in Tab. 13, with the stiffness moment and rotation

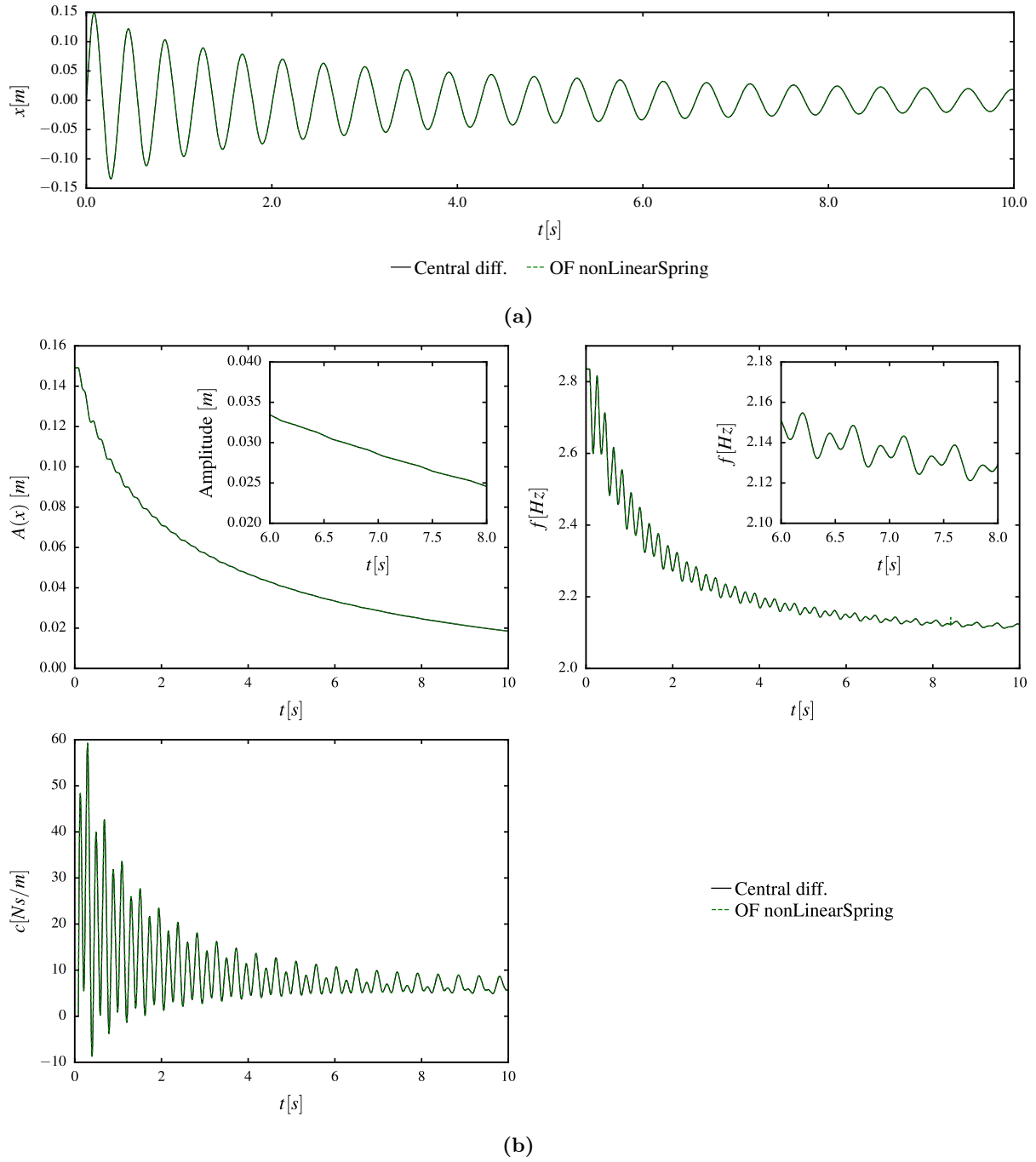


Figure 8. Translational nonlinear stiffness and nonlinear damping dynamic system response: central finite differences vs. `nonLinearSpring` library. a) Time-dependent free damped vibration, b) Time dependent oscillation amplitude (top-left), frequency (top-right) and damping (bottom-left).

following the relationship in Fig. 9. In this case, the stiffness (the slope of the curve) at zero rotation is equal to 8000.0 Nm/rad. Fig. 10a shows the solution provided by the `nonLinearAxialAngularSpring` library and the central finite differences numerical solution of the dynamic equilibrium equation. Both approaches deliver the same solution. This verifies the nonlinear damping implementation for the rotational DOF. As in the former case, the changes in stiffness (frequency) and damping over time are evident in Fig. 10b.

Table 13. Mechanical properties of the rotational single DOF nonlinear system with stiffness defined by a list of values and Rayleigh model for damping. I is the Inertia of the system, c_1 is the linear constant of the Rayleigh model, c_2 is the nonlinear constant of the Rayleigh model, α_0 is the initial angular position of the system and $\dot{\alpha}_0$ is the initial angular velocity of the system.

I [kgm^2]	c_1 [Nms/rad]	c_2 [Nms^3/rad^3]	α_0 [rad]	$\dot{\alpha}_0$ [rad/s]
40.6452	20.0	50.0	0.0	2.65

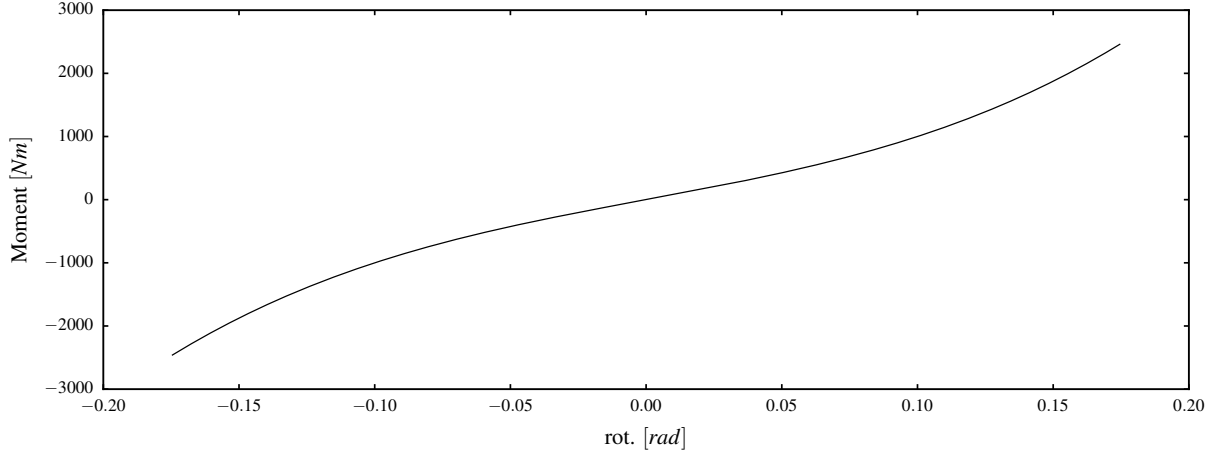


Figure 9. Stiffness moment vs. rotation.

The number of time steps, for the considered time step size, for resolving the first cycle of oscillation for the translational and rotational spring cases reported above are presented in Tab. 14.

Table 14. Number of time steps needed for solving the first oscillation cycle for the translational and rotational springs with nonlinear stiffness and nonlinear damping.

Δt [s]	$n(\Delta t)$	
	Translational	Rotational
1.0×10^{-6}	362113	403446

5. FSI Application examples: wingMotion OpenFOAM® tutorial

In the previous section, the implementation of the newly developed libraries for solving the dynamic response of nonlinear systems was numerically verified. In this section, the wingMotion OpenFOAM® tutorial is selected to demonstrate the capabilities of the `nonLinearSpring` and `nonLinearAxialAngularSpring` to efficiently solve FSI problems involving nonlinear dynamic systems. In the application cases reported next, the results obtained with the new libraries are compared with the original standard implementation on OpenFOAM® for the linear case, and with the response provided by a partitioned approach that combines OpenFOAM®, as fluid solver, and CalculiX, as structural solver, using preCICE, as coupling tool. This is a set of relatively simple application cases, adopting a 2D URANS approach and the $k - \omega$ two-equation turbulence model. The wingMotion tutorial can be found inside the OpenFOAM® installation folder at the following relative path: `$FOAM-TUTORIALS/incompressible/pimpleFoam/RAS/wingMotion`

5.1. wingMotion tutorial: linear stiffness and linear damping in the heave DOF.

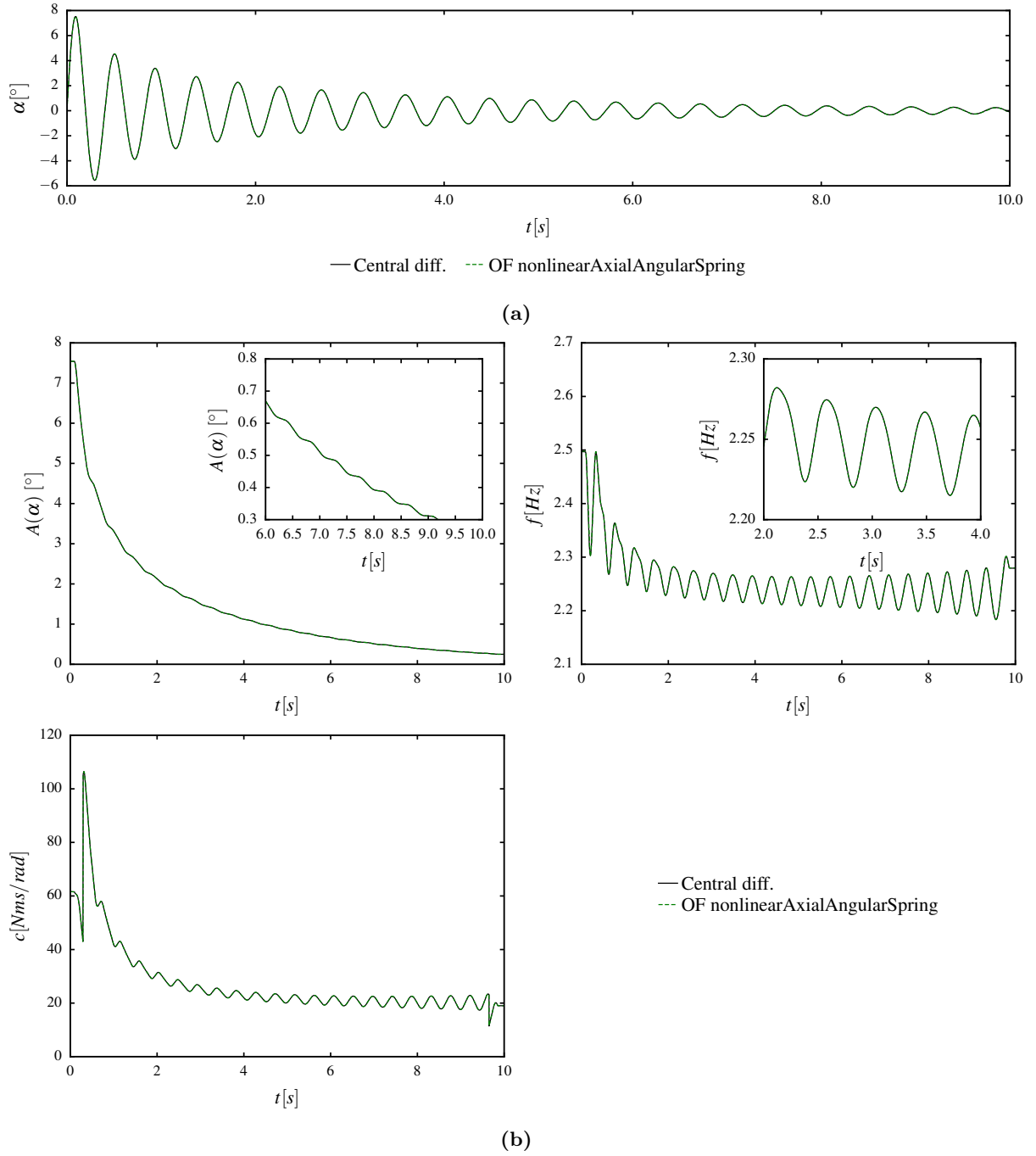


Figure 10. Rotational nonlinear stiffness and nonlinear damping dynamic system response: central finite differences vs. `nonLinearAxialAngularSpring` library. a) Time-dependent free damped vibration, b) Time dependent oscillation amplitude (top-left), frequency (top-right) and damping (bottom-left).

In this first application case, the only allowed movement of the airfoil is in the heave DOF and the dynamic properties of the system are the ones reported in Tab. 2, except for the initial conditions in displacement and velocity of the system, which are both set to zero, and the value of the viscous damping, which is set to 2.0 [Ns/m]. The velocity of the incoming smooth flow is $U = 100.0$ m/s and the chord of the airfoil is $C = 1.0$ m.

As a preliminary task, a sensitivity study depending on the size of the time step in the step-by-step method for solving the dynamic equilibrium equation was conducted with the purpose of confirming for

this FSI problem the results obtained in section 4 for the free vibration cases (see Fig. 11). It was observed that both 1.0×10^{-5} s and 1.0×10^{-6} s time steps provide the same results. Based on this study, the remaining cases for the wingMotion tutorial that are reported next have been solved adopting a time step of 1.0×10^{-5} s.

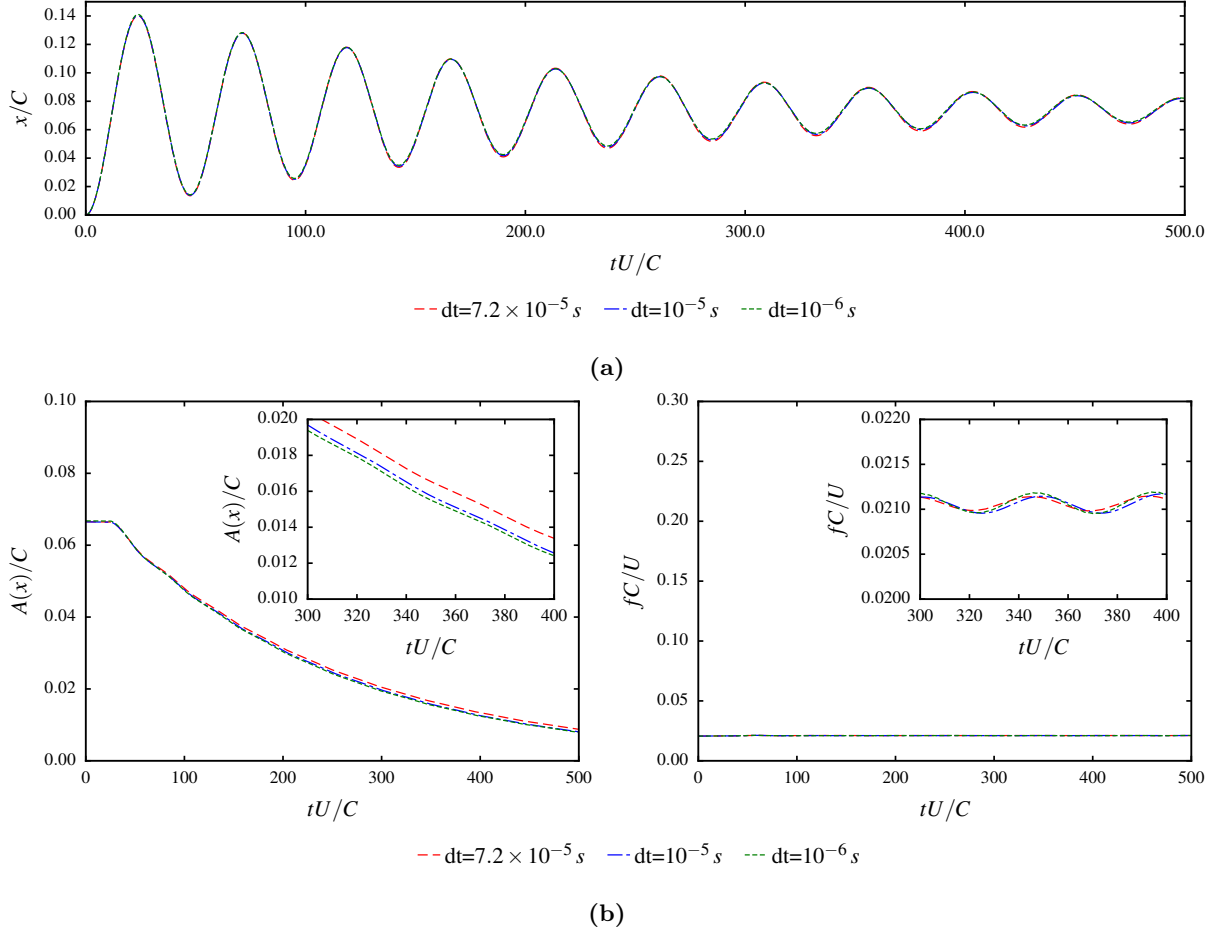


Figure 11. Sensitivity study of the linear case under heave oscillations and wind conditions: a) Time-histories of the oscillations and b) Time evolution of the amplitudes (left) and frequencies (right).

The time-histories of the lift coefficient and the heave oscillations for the heave linear case are presented in Fig. 12a for the standard `linearSpring` library on OpenFOAM[®], the newly developed `nonLinearSpring` and preCICE. The results provided by OpenFOAM[®] with both the standard `linearSpring` and the new `nonLinearSpring` libraries were almost identical. These simulations with OpenFOAM[®] resulted in slightly larger value for the amplitude of oscillation over time than the simulation with preCICE. These minor differences may be expected due to intrinsic uncertainties in the transfer of the interface data between the fluid and the structural solver using preCICE. This can be observed only in Fig. 12b where small differences in the amplitudes of oscillation can be observed in the zoomed image. On the other hand, no differences in the frequency response were identified.

5.2. wingMotion tutorial: nonlinear stiffness and linear damping in the heave DOF.

The same application case has also been analysed considering a nonlinear relationship between displacements and spring forces, specifically the relationship depicted in Fig. 13, which features a stiffness (slope of the curve) equivalent to the fully linear case in subsection 5.1, previously introduced, at zero

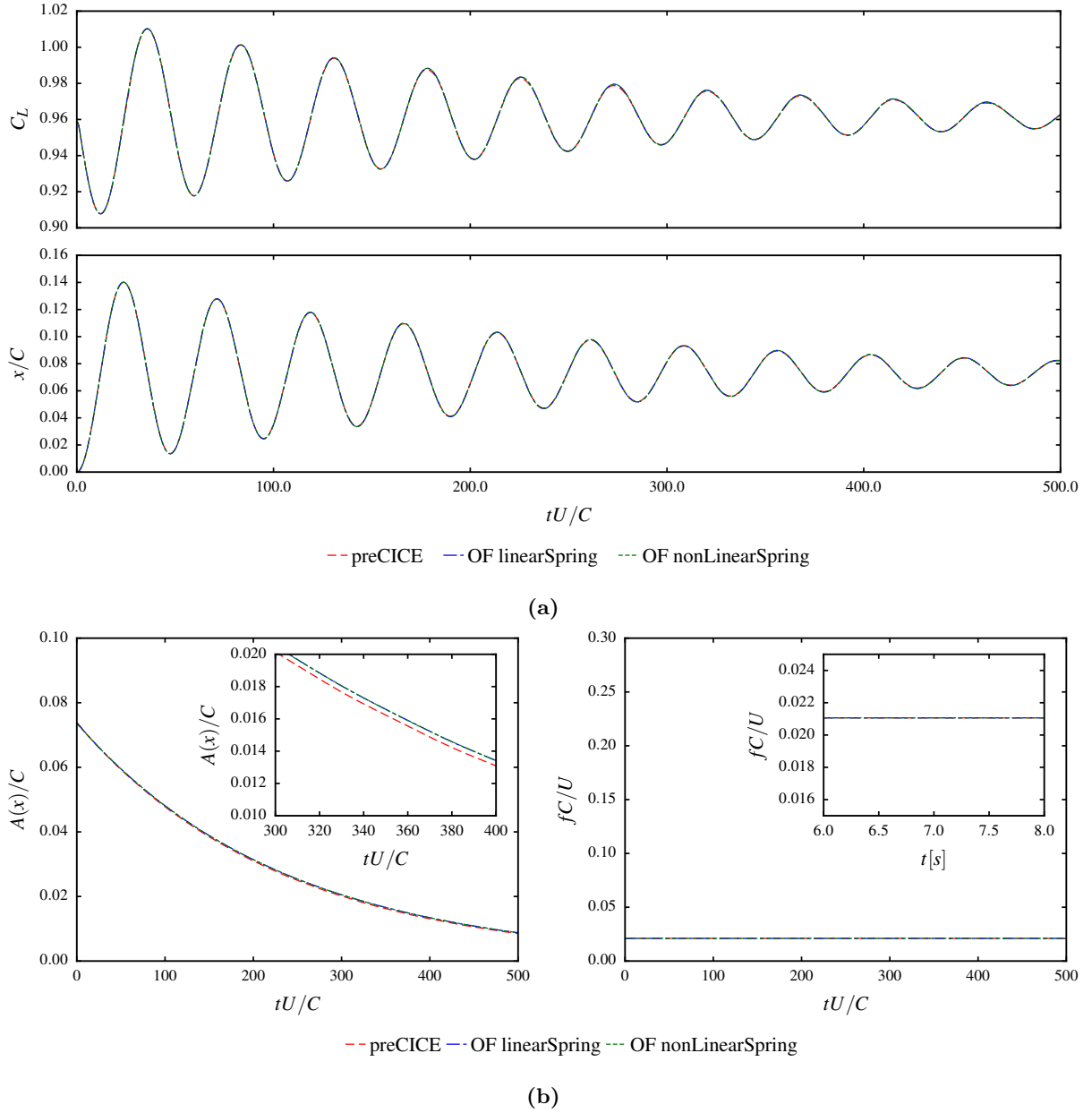


Figure 12. wingMotion tutorial heave DOF linear stiffness and linear damping response: `linearSpring`, vs. `nonLinearSpring` vs. `preCICE` partitioned approach. a) Time-dependent lift coefficient (top) and non-dimensional amplitude of oscillation (bottom) and b) Time dependent non-dimensional oscillation amplitude (left) and non-dimensional frequency (right).

displacement.

The results for the flow-induced oscillation in heave obtained using the `nonLinearSpring` library and the standard `linearSpring` library on OpenFOAM® coupled with Calculix using `preCICE` are presented in Fig. 14. It can be seen that both approaches yielded the same results, presenting only negligible differences in the amplitude of oscillation, that were translated also into minor differences in the frequency of oscillation due to the nonlinear stiffness of the system. In the charts, the results obtained using the `nonLinearSpring` library with time steps 1.0×10^{-5} and 1.0×10^{-6} are also reported to demonstrate the insensitivity of the results with the time-step size. The results for the `preCICE` simulation were obtained

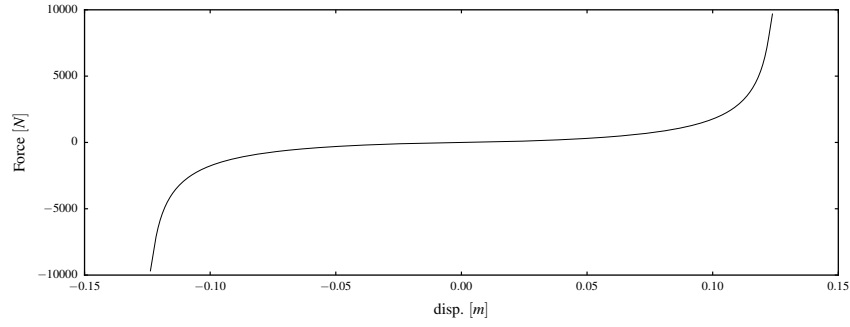


Figure 13. Nonlinear spring force-displacement relation.

considering a time step of 1.0×10^{-5} s.

5.3. wingMotion tutorial: nonlinear stiffness and nonlinear damping in the heave DOF.

The same application case has been solved with the `nonLinearSpring` library considering a nonlinear spring and nonlinear damper system with the properties shown in Tab. 15. Three different time steps were considered to assess the sensitivity of the solution in terms of accuracy and stability. The three time steps delivered time-histories that were very well aligned (see Fig. 15).

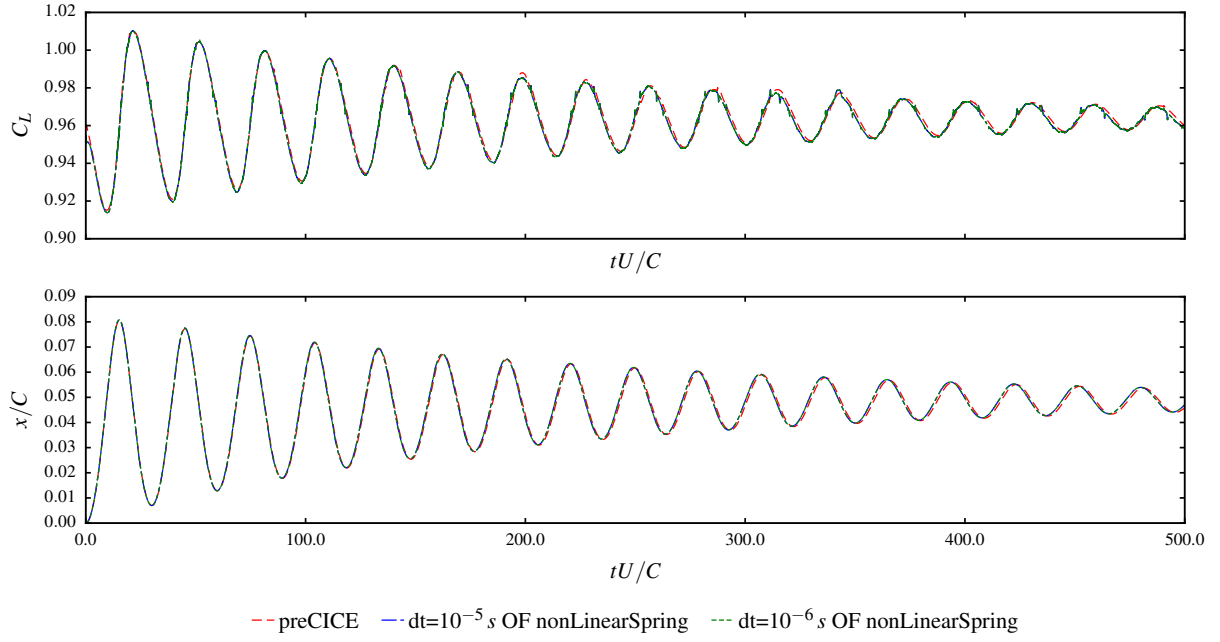
Table 15. Mechanical properties of the nonlinear single DOF system with Duffing model for stiffness and Mann model for damping under wind conditions. m is the mass of the system, A is the linear spring constant of the Duffing model, B is the nonlinear spring constant of the Duffing model, c_1 is the linear constant of the Mann model, c_2 is the nonlinear constant of the Mann model, x_0 is the initial position of the system and $\dot{x}_0$ is the initial velocity of the system.

m [kg]	A [N/m]	B [N/m ³]	c_1 [Ns/m]	c_2 [Ns ³ /m ³]	x_0 [m]	$\dot{x}_0$ [m/s]
22.9	4000.0	200000.0	6.0	5000.0	0.0	2.65

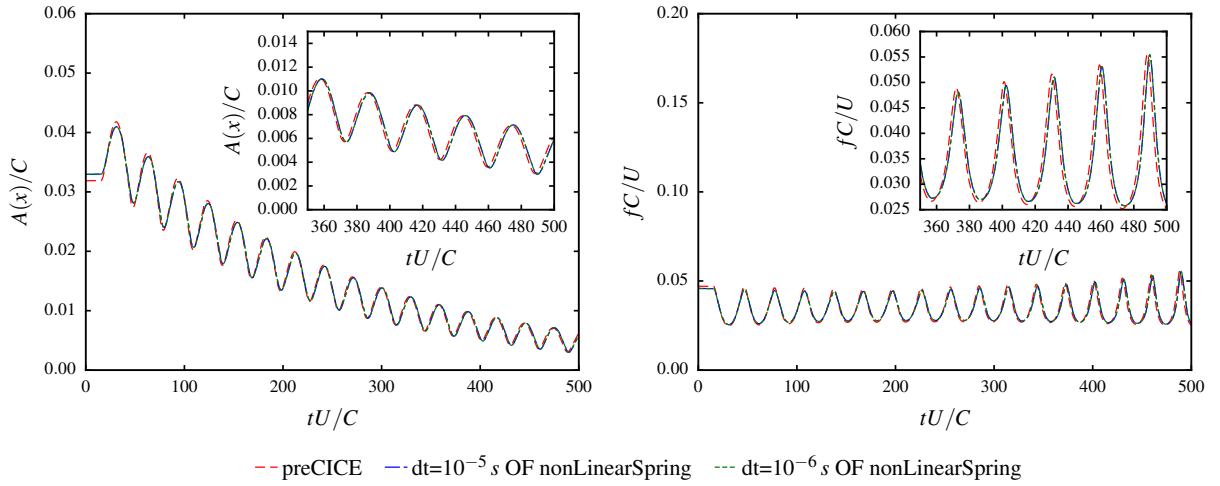
The complete set of results, for a time step of 1.0×10^{-5} s and including the evolution of damping over time, is reported in Fig. 16.

5.4. wingMotion tutorial: nonlinear stiffness and nonlinear damping in the pitch DOF.

In this application case, the only allowed movement of the airfoil is the pitch rotation. The one DOF nonlinear stiffness and nonlinear damping system has the same properties as the one in subsection 4.3 for the rotational DOF, and the velocity of the incoming smooth flow is 100.0 m/s. The results applying the newly developed `nonlinearAxialAngularSpring` library are presented in Fig. 17. The figures show the evolution over time of the system's amplitude and frequency of oscillation, as well as the damping. It can be noted that the moment coefficient timeseries differed from the typical sinusoidal shape signal as it was variably damped over time (bottom left chart in Fig. 17b).



(a)



(b)

Figure 14. wingMotion tutorial heave DOF nonlinear stiffness and linear damping response: nonLinearSpring vs. preCICE partitioned approach. a) Time-dependent lift coefficient (top) and non-dimensional amplitude of oscillation (bottom) and b) Time dependent non-dimensional oscillation amplitude (left) and non-dimensional frequency (right).

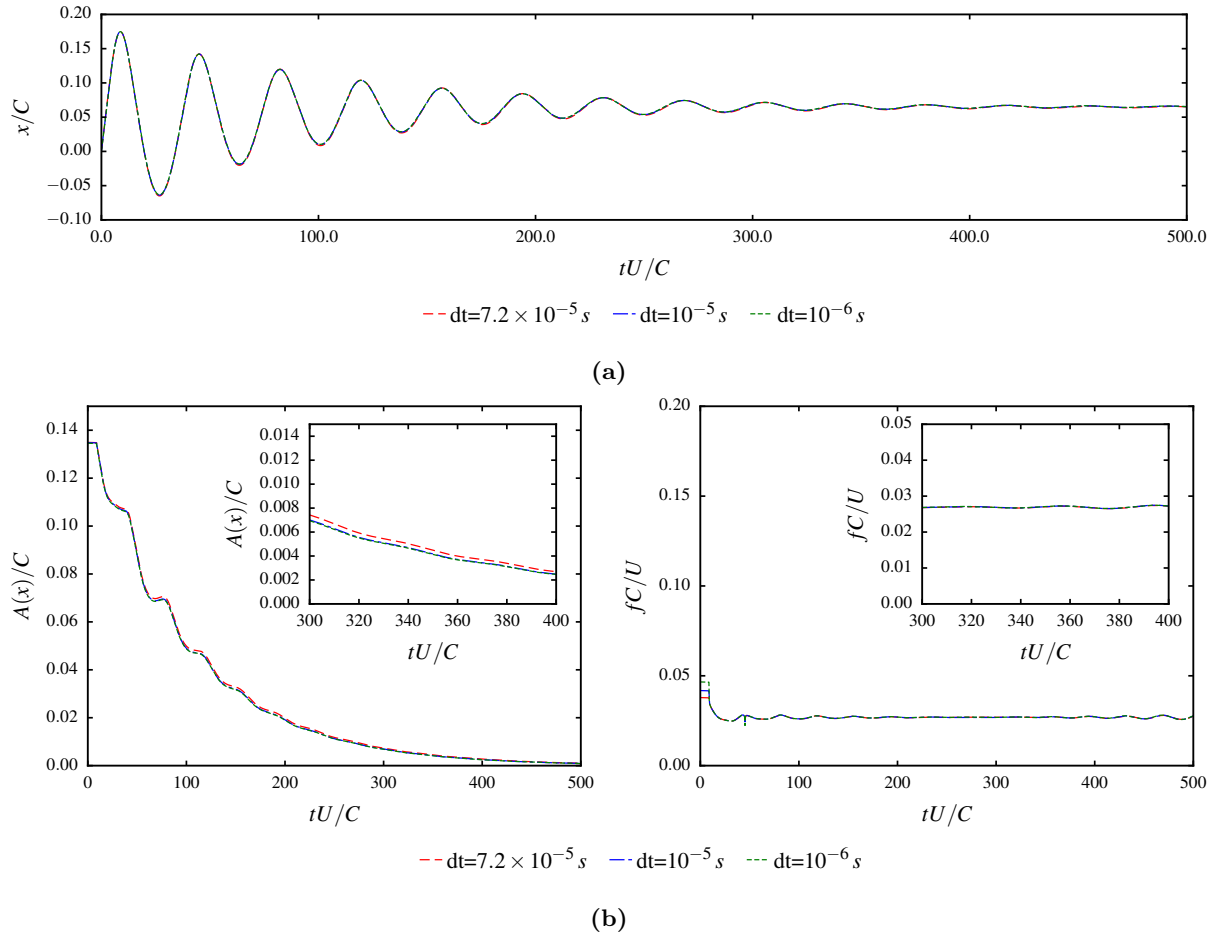
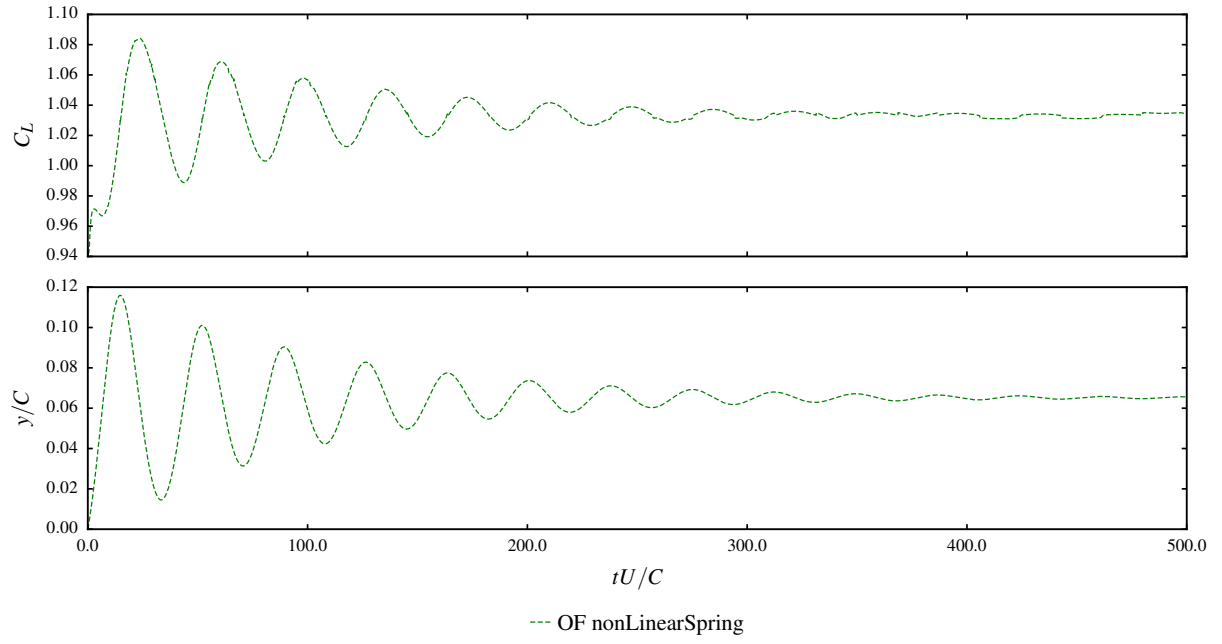
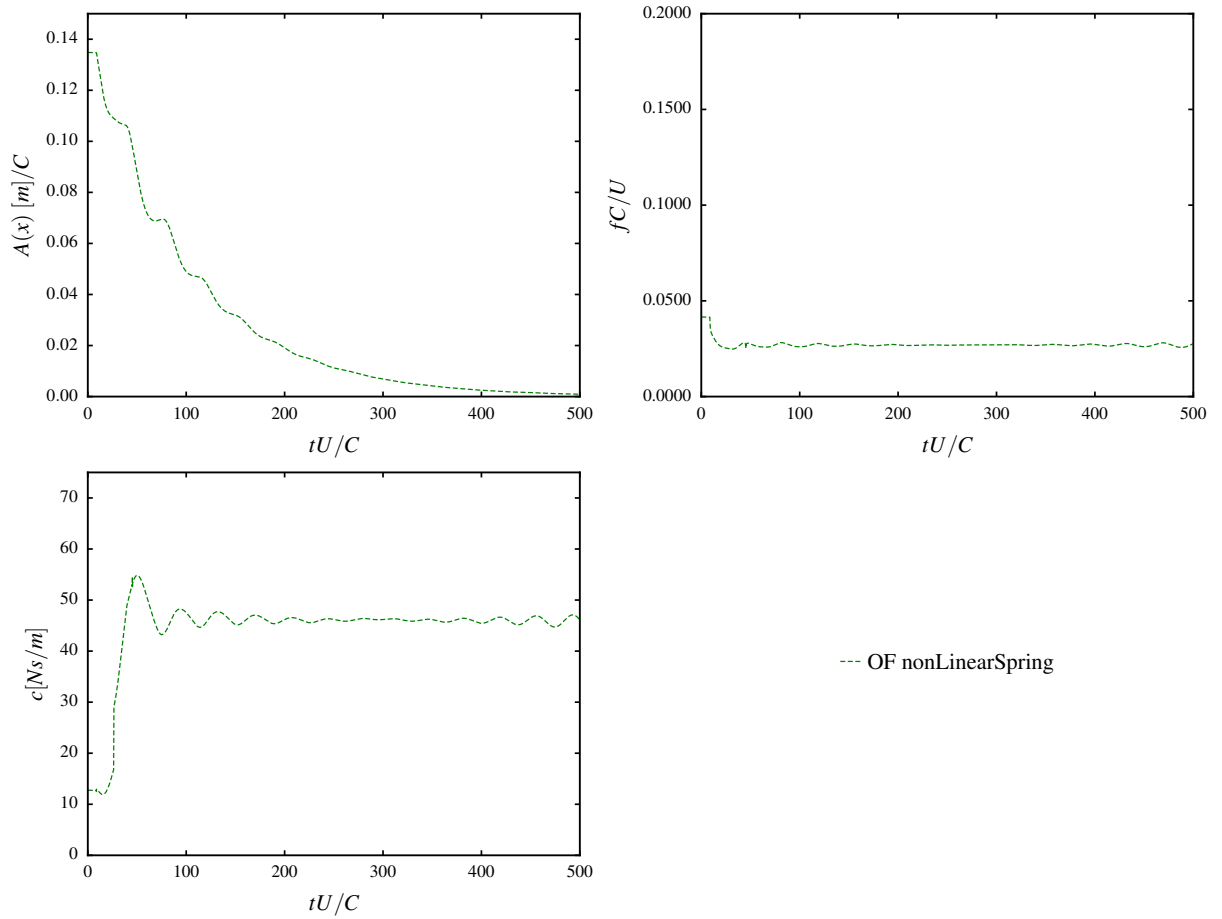


Figure 15. Temporal sensitivity of the nonlinear case under heave oscillation and wind conditions. a) Time-dependent non-dimensional oscillations and b) Time dependent non-dimensional oscillation amplitude (left) and non-dimensional frequency (right).



(a)



(b)

Figure 16. wingMotion tutorial heave DOF nonlinear stiffness and nonlinear damping response solved with the `nonLinearSpring` library. a) Time-dependent lift coefficient (top) and non-dimensional heave oscillation (bottom) and b) Time dependent non-dimensional heave oscillation amplitude (top-left), non-dimensional frequency (top-right) and damping (bottom-left).

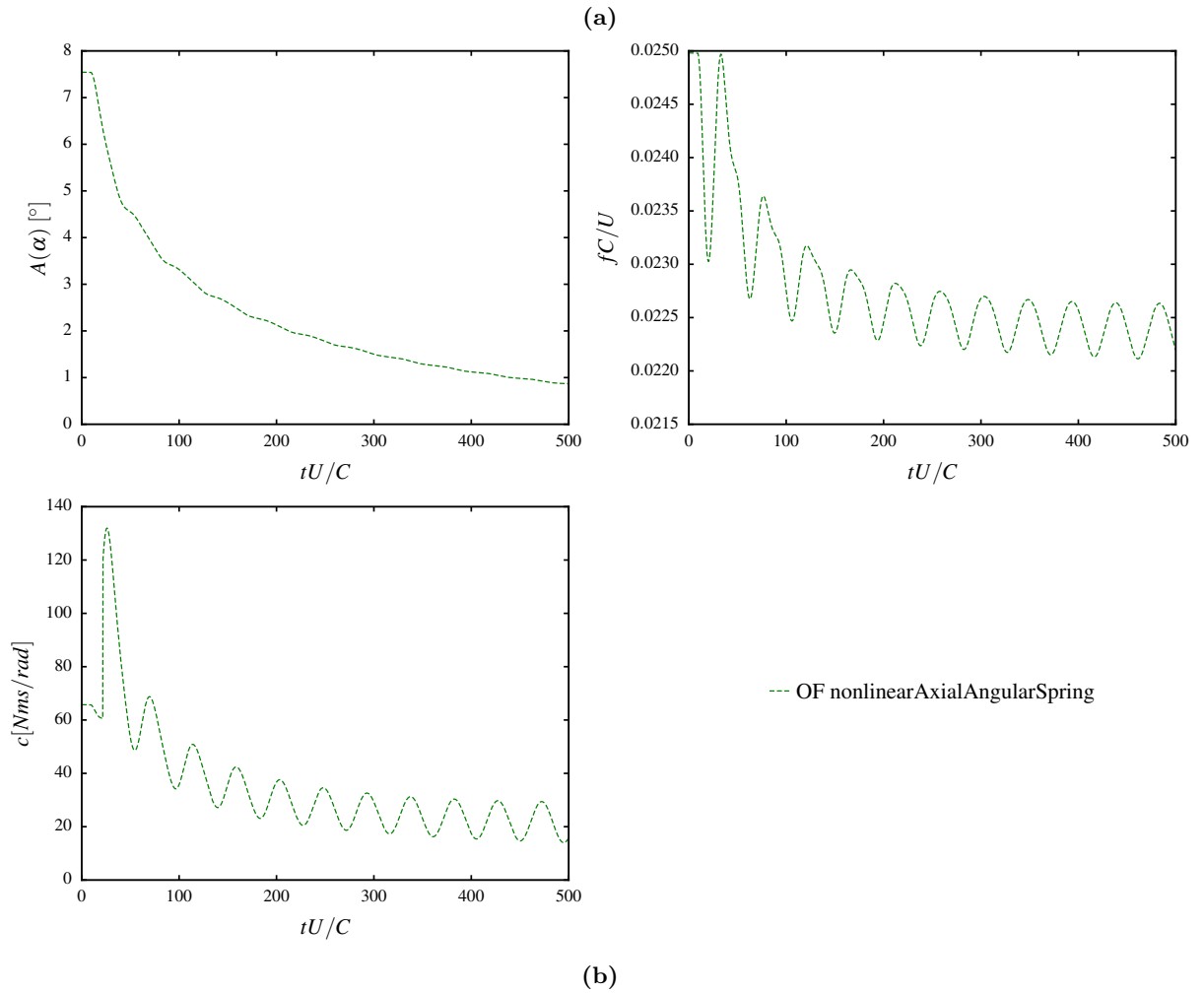
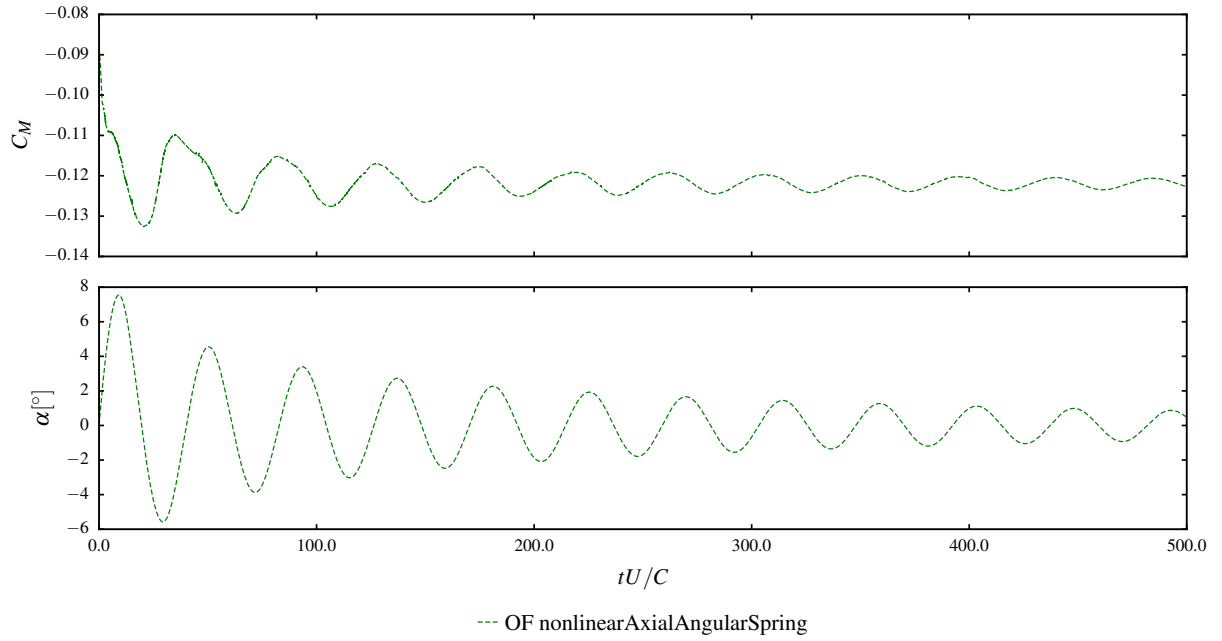


Figure 17. wingMotion tutorial pitch DOF nonlinear stiffness and nonlinear damping response solved with the `nonLinearAxialAngularSpring` library. a) Time-dependent moment coefficient (top) and rotational oscillation (bottom) and b) Time dependent non-dimensional pitch oscillation amplitude (top-left), non-dimensional frequency (top-right) and damping (bottom-left).

6. FSI application example: Galloping response of a 3:2 rectangular prism featuring nonlinear stiffness

The galloping response of a bluff body featuring nonlinear stiffness represents a more complex application example that was solved here by implementing the newly developed libraries for nonlinear dynamic systems along with a 3D LES approach combined with a high density mesh resulting in a state-of-the-art highly computationally demanding case.

In this application case a prism of width to height ratio 3:2, modeled as a nonlinear single DOF system in heave at 0° angle of attack was studied under incoming smooth flow conditions for which it should be experiencing galloping oscillations. The simulations were conducted at a reduced wind speed $U^* = U/(f_0 H) = 31.0$, where U represents the incoming wind speed [m/s], f_0 is the frequency associated with the stiffness of the nonlinear spring at zero displacement [Hz] and H is the height of the prism. This reduced velocity corresponds with a Reynolds number $Re_H = UH/\mu = 143000$, with μ the kinematic viscosity of air. The mechanical properties of the one DOF system are provided in Tab. 16, while the force-displacement relationship of the nonlinear spring, following the formulation provided in Kluger et al. [2, 15], is the one shown in Fig. 2. Linear interpolation was used for calculating the restoring force from the provided force-displacement relationship.

Table 16. Mechanical properties of the single DOF system for the 3:2 prism case. m is the mass of the system, k_0 is the spring stiffness at zero displacement, c is the viscous damping of the system, y_0 is the initial position of the system and $\dot{y}_0$ is the initial velocity of the system.

m [kg]	k_0 [N/m]	c [Ns/m]	y_0 [m]	$\dot{y}_0$ [m/s]
290.25	1040.399	4.594	0.0	0.18

The simulations were conducted using a 3D LES approach, by means of the Yoshizawa’s model [26]. The finite volumes flow domain mesh was generated using the snappyHexMesh application, yielding a total of 1925744 cells. The fluid domain has a total length of $50H$, with the prism located at $17.25H$ from the inlet. The total height of the fluid domain is $34H$, being the prism equidistant between the upper and lower surfaces. The fluid domain has a spanwise dimension equal to $4H$. The mesh sizes for the different regions defined in the flow domain can be found in Tab. 17, and the properties of the boundary layer are presented in Tab. 18. Symmetric boundary conditions have been used in its lateral, upper and lower surfaces. Second order schemes have been selected for performing the simulations, and the selected schemes are presented in Tab. 19.

Table 17. Mesh regions and element sizes.

Region	Lower left corner		Upper right corner		Element size
	x/H	y/H	x/H	y/H	
1	-17.25	-17.00	32.75	17.00	H
2	-10.00	-10.00	16.33	10.00	$H/2$
3	-4.50	-4.50	17.14	4.50	$H/2^2$
4	-3.25	-3.25	9.75	3.25	$H/2^3$
5	-2.25	-2.25	2.25	2.25	$H/2^4$
6	-1.25	-1.00	1.25	1.00	$H/2^5$
7	-0.75	-0.50	0.75	0.50	$H/2^6$

Images of the mesh identifying the different regions and mesh characteristics are shown in Fig. 18. The time-histories of the lift coefficient (Fig. 19) and heave oscillations (Fig. 20) for the simulations

with the new implemented library in OpenFOAM `nonLinearSpring`, and the coupling of OpenFOAM and Calculix with preCICE presented slight differences among them. Considering the complete CFD model implementation in the two approaches, some uncertainties exist about the specific implementation of the routines, numerical procedures and values in multiple parameters. Furthermore, referring to the preCICE solution, the information about the displacement of the prism, allocated in the interface region, must be transferred from OpenFOAM to CalculiX through preCICE, which is affected by truncation errors. Consequently, some minor differences are expected to occur between the two simulations, yielding differences in specific values (the lift force, for example) for a specific time point, although the overall phenomenon is correctly simulated by the two approaches. In the tables and figures that are presented next, the differences between the two approaches in outputs like the standard deviation of the oscillations and the dominant frequency of the lift coefficient are marginal. A more in-depth analysis focusing on the standard deviation of the converged heave oscillations and the main frequency peak of the heave oscillations, presented in Tab. 20, provided almost the same results, meaning that both approaches were reproducing the same phenomenon with the same level of fidelity. It must be noted that the relative differences in the values obtained from the OpenFOAM[®] new library and preCICE for the mean values of the non-dimensional oscillations and lift coefficient are larger than the ones obtained for standard deviations. This is due to the very low values close to zero of the mean values. Further comparisons between the two cases are provided in Figs. 21 and 22.

Table 18. Boundary layer properties. (h_1 is the height of the first layer of the boundary layer, gr_{bl} is the growth ratio of the boundary layer and n_{bl} is the number of layers forming the boundary layer)

h_1/H	gr_{bl}	n_{bl}
1.12×10^{-3}	1.35	9

Table 19. Numerical Schemes used in the 3:2 prism simulations.

Numerical Schemes			
Scheme	Method	Scheme	Method
temporal	backward	gradients	Gauss linear
divergence (convection)	LUST/Gauss linear	laplacian	Gauss linear corrected

The monolithic approach based on the `nonLinearSpring` library enabled a faster model execution relative to the coupling between OpenFOAM[®] and CalculiX, to a large extent due to the more demanding communication requirements between the two independent solvers linked through preCICE in the partitioned approach. The number of CPU hours using 32 cores required for advancing 1 second of simulation is 1.0 with `nonLinearSpring` and 1.3 with preCICE. Finally, in Fig. 23, the Q criterion for level 1.0 at non-dimensional time $tU/H = 3197.65$ and non-dimensional amplitude of oscillation $y/H = 0.2327$ is presented, depicting the complexity of the flow in the FSI problem.

Table 20. Oscillations and lift coefficient properties for the two applications of the ratio 3:2 rectangular prism.

	$\bar{y}/H$	$\sigma(y)/H$	$f_{S_{peak}(y)}$	$\overline{C_L}$	$\sigma(C_L)$	$f_{S_{peak}(C_L)}$
preCICE	6.093×10^{-4}	0.166	0.066	-1.000×10^{-4}	1.283	0.066
OpenFOAM [®] new library	6.762×10^{-5}	0.167	0.067	-9.700×10^{-3}	1.491	0.067

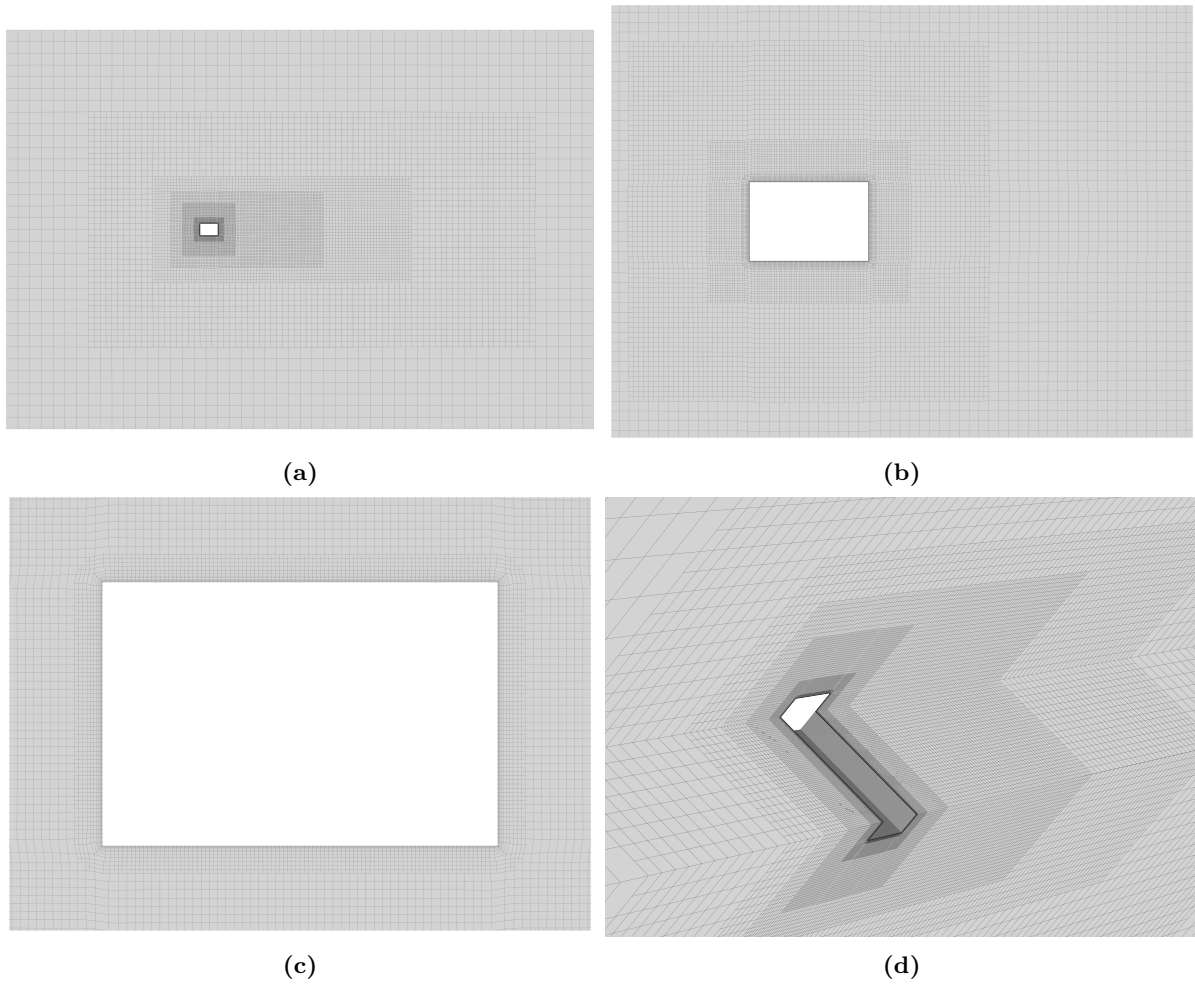


Figure 18. Different images of the mesh for the 3:2 prism: a) overall mesh, b) detail of the prism and wake, c) detail of the prism, d) mesh sections showing the spanwise distribution of cells.

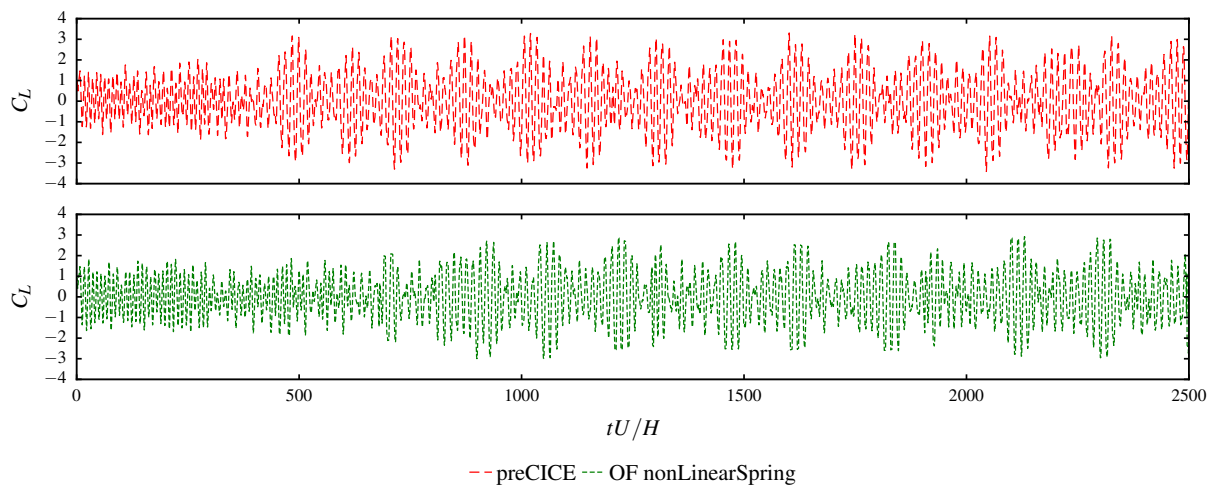


Figure 19. Lift coefficient time-histories of the 3:2 prism with preCICE (top) and the nonLinearSpring library (bottom). U is the incoming wind speed and H is the height of the prism.

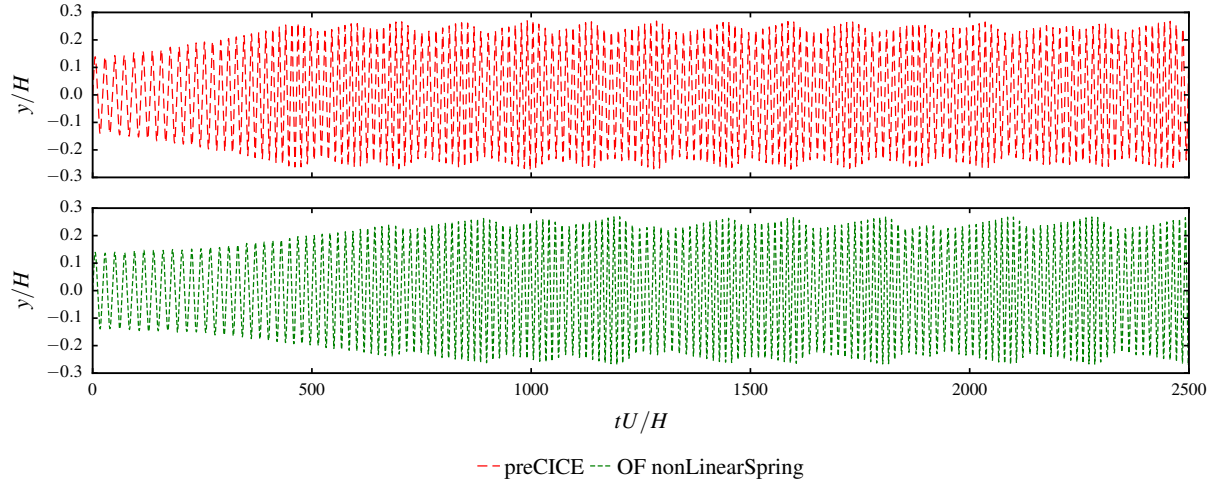


Figure 20. Non-dimensional heave oscillation time-histories of the 3:2 prism with preCICE (top) and the nonLinearSpring library (bottom). H is the height of the prism.

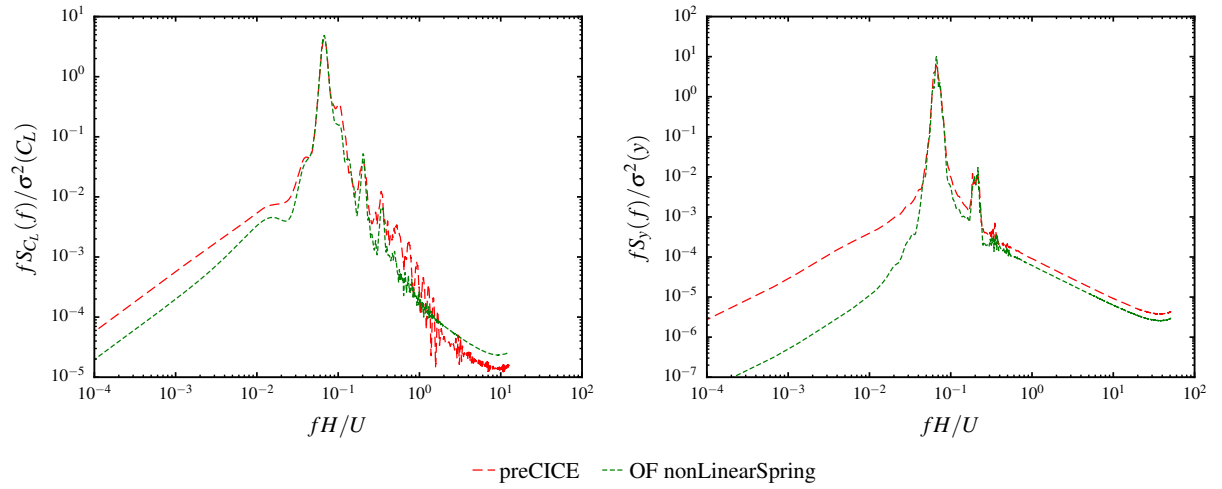


Figure 21. Heave oscillation and lift coefficient spectra.

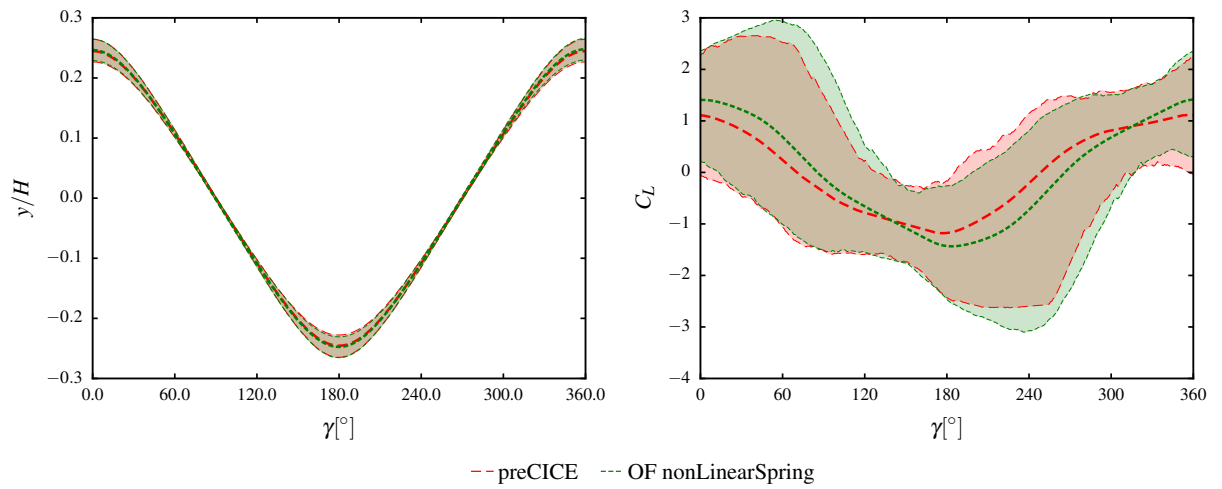


Figure 22. Phase averaged heave oscillation and lift coefficient. The thicker line in the middle of the shaded area represents the phase averaged mean, while the limits of the shaded area represents the lower and upper bound of a 80% confidence level.

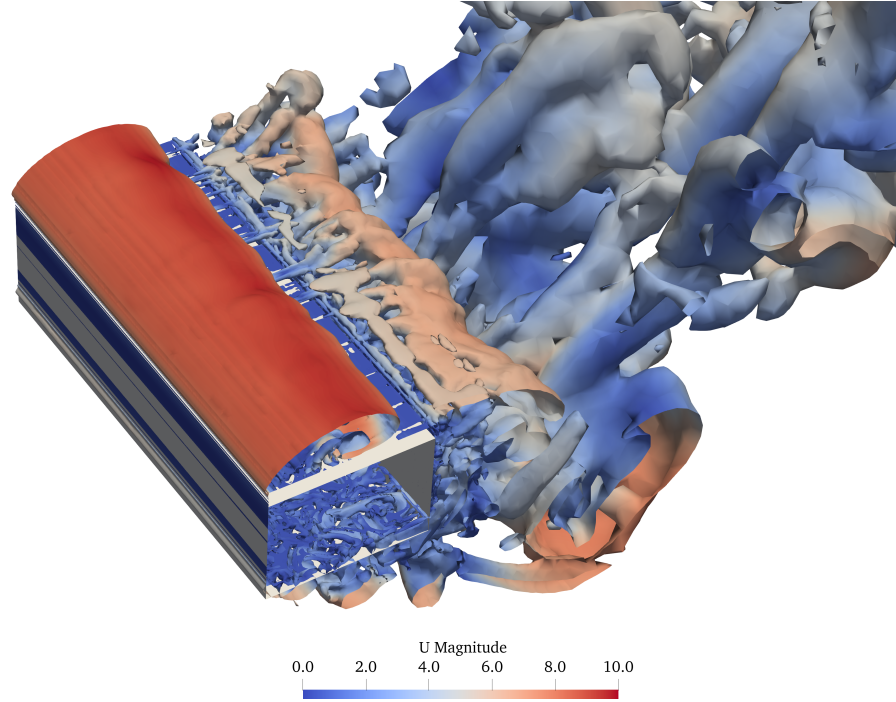


Figure 23. Q criterion contour at level 1.0 for $tU/H = 3197.65$ and $y/H = 0.2327$.

7. Conclusion

The present study introduced nonlinear spring–damper libraries for the OpenFOAM® solver within the 6-DOF rigid body motion framework that significantly enhanced the platform’s ability to simulate fluid–structure interaction problems involving nonlinear structural dynamical systems. By extending the existing linear restraints to include nonlinear translational and rotational springs and dampers, the implementation enables users to prescribe restoring and dissipative forces either adopting analytical models such as Duffing stiffness, Rayleigh damping, or Mann-type formulations, or through user-defined interpolation tables for the nonlinear properties of the dynamical system. This flexibility allows for a seamless transition between linear models and complex nonlinear ones, broadening the scope of rigid-body 6-DOF structural systems that can be studied within the same solver environment adopting a monolithic approach for FSI simulations.

Structural verification against established approaches, including analytical models, central difference schemes and OpenFOAM® – CalculiX coupled via preCICE, confirmed the reliability of the libraries. In both linear and nonlinear cases, the new implementation reproduced the expected oscillatory dynamics with only minor differences in amplitude, while maintaining accurate frequency predictions. Importantly, the proposed implementation eliminated the need for external structural solvers for systems modelled as 6-DOF rigid systems, reducing computational cost and simplifying the setup process.

The structural verification and FSI demonstration examples highlighted the importance of adopting time steps that are small enough to yield accurate and stable numerical solutions for nonlinear cases. Additionally, it must be noted that for the tabulated input of stiffness and damping relationships a relatively large number of points is required for their reliable definition, with the dynamic response results being sensitive to the approach implemented for interpolation and extrapolation.

The application cases demonstrated the practical value of the libraries in resolving aeroelastic phenomena of increasing complexity. Simulations of airfoils oscillating in the heave and pitch DOF highlighted the ability to capture wind-induced oscillations with nonlinear stiffness and damping, while the challenging 3D LES 3:2 prism case delivered the galloping response under incoming smooth flow with fidelity levels comparable to the coupled partitioned approach. These results underscore the capability of the libraries to provide insights into rigid body nonlinear mechanisms relevant to energy harvesting, vibration control, and aeroelastic stability.

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References

- [1] G. Hou, J. Wang, and A. Layton, "Numerical methods for fluid–structure interaction — a review," *Communications in Computational Physics*, vol. 12, no. 2, pp. 337–377, 2012.
- [2] J. M. Kluger, T. P. Sapsis, and A. H. Slocum, "Robust energy harvesting from walking vibrations by means of nonlinear cantilever beams," *Journal of Sound and Vibration*, vol. 341, pp. 174–194, 4 2015.
- [3] X. Tang and L. Zuo, "Simulation and experiment validation of simultaneous vibration control and energy harvesting from buildings using tuned mass dampers," in *Proceedings of the American Control Conference*, 2011, pp. 3134–3139.
- [4] A. Trimble, "Energy harvesting of random wide-band vibrations with applications to an electro-magnetic rotational energy harvester," Ph.D. Thesis, Massachusetts Institute of Technology, 2011.
- [5] Q. Li, C. Wang, C. Liu, Z. Li, X. Iiu, and L. He, "Development trend of nonlinear piezoelectric energy harvesters," *Journal of Electronic Materials*, vol. 54, pp. 1–23, 2025.
- [6] O. V. Gendelman, T. Sapsis, A. F. Vakakis, and L. A. Bergman, "Enhanced passive targeted energy transfer in strongly nonlinear mechanical oscillators," *Journal of Sound and Vibration*, vol. 330, no. 1, pp. 1–8, 1 2011.
- [7] T. P. Sapsis, A. F. Vakakis, O. V. Gendelman, L. A. Bergman, G. Kerschen, and D. D. Quinn, "Efficiency of targeted energy transfers in coupled nonlinear oscillators associated with 1:1 resonance captures: Part ii, analytical study," *Journal of Sound and Vibration*, vol. 325, no. 1–2, pp. 297–320, 8 2009.
- [8] B. Ando, S. Baglio, V. Marletta, and A. R. Bulsara, "A comparison of linear and non-linear strategies for energy harvesting from mechanical vibrations," *Frontiers in Physics*, vol. 10, no. 1032978, 2023.
- [9] Y. Wu, Z. Cheng, R. McConkey, F.-S. Lien, and E. Yee, "Modelling of flow-induced vibration of bluff bodies: A comprehensive survey and future prospects," *Energies*, vol. 15, no. 22, p. 8719, 11 2022.
- [10] S. Jamshidi, A. Haghighi Poshtiri, and M. E. Maali, "Numerical simulation of flow-induced vibration of the one-degree-of-freedom circular cylinder using random vortex-boundary element method at turbulent flow," *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 45, no. 125, 2023.
- [11] G. Chourdakis, D. Schneider, and B. Uekermann, "Openfoam-precice: Coupling openfoam with external solvers for multi-physics simulations," *OpenFOAM Journal*, vol. 3, no. 88, pp. 1–25, 2023.
- [12] G. Chourdakis *et al.*, "precice v2: A sustainable and user-friendly coupling library," *Open Research Europe*, vol. 2, p. 51, 9 2022.
- [13] Z. Zhou, Y. Wen, C. Cai, and Q. Zeng, *Fundamentals of Structural Dynamics*. Changsha, China: Central South University Press, published by Elsevier Inc., 2021.
- [14] E. Halvorsen and S. D. Nguyen, "Mems electrostatic energy harvesters with nonlinear springs," in *Advances in Energy Harvesting Methods*, N. Elvin and A. Erturk, Eds. New York, NY, USA: Springer, 2013, pp. 63–90.
- [15] J. M. Kluger, T. P. Sapsis, and A. H. Slocum, "A high-resolution and large force-range load cell by means of nonlinear cantilever beams," *Precision Engineering*, vol. 43, pp. 241–256, 1 2016.
- [16] S. Timoshenko, *Strength of Materials*. New York: Van Nostrand, 1930.
- [17] T. Al-hababi, M. Cao, B. Saleh, N. F. Alkayem, and H. Xu, "A critical review of nonlinear damping identification in structural dynamics: Methods, applications, and challenges," *Sensors*, vol. 20, no. 24, p. 7303, 12 2020.
- [18] S. J. Elliott, M. Ghandchi Tehrani, and R. S. Langley, "Nonlinear damping and quasi-linear modelling," *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 373, no. 2051-20140402, 9 2015.
- [19] M. Ghandchi Tehrani and S. J. Elliott, "Extending the dynamic range of an energy harvester using nonlinear damping," *Journal of Sound and Vibration*, vol. 333, pp. 623–629, 2 2014.
- [20] J. W. S. Rayleigh, *The Theory of Sound*, 2nd ed. Mineola, NY, USA: Dover Publications Inc., 1945, section 68a.
- [21] B. P. Mann, "Broadband energy harvesting from a bistable potential well," in *Advances in Energy Harvesting Methods*, N. Elvin and A. Erturk, Eds. New York, NY, USA: Springer, 2013, pp. 91–115.
- [22] G. C. Hart and K. K. F. Wong, *Structural Dynamics for Structural Engineers*. New York, NY, USA: John Wiley & Sons, Inc., 2000.
- [23] S. Y. Chang, "Studies of newmark method for solving nonlinear systems: (i) basic analysis," *Journal of the Chinese Institute of Engineers*, vol. 27, no. 5, pp. 651–662, 2004.
- [24] —, "Studies of newmark method for solving nonlinear systems: (ii) verification and guideline," *Journal of the Chinese Institute of Engineers*, vol. 27, no. 5, pp. 663–675, 2004.
- [25] G. Gao and L. Zhu, "Nonlinearity of mechanical damping and stiffness of a spring-suspended sectional model system for wind tunnel tests," *Journal of Sound and Vibration*, vol. 355, pp. 369–391, 2015.
- [26] A. Yoshizawa, "Statistical theory for compressible turbulent shear flows, with the application to subgrid modeling," *Physics of Fluids*, vol. 29, no. 7, pp. 2152–2164, 7 1986.